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Application of Structural Analysis to Improve Fault Diagnosis in a Gas Turbine

Cristina Verde and Marino Sánchez-Parra
Instituto de Ingeniería, Universidad Nacional Autónoma de Mexico
Instituto de Investigaciones Eléctricas
México

1. Introduction

This chapter deals with the fault diagnosis issues for a Gas Turbine, *GT*, of a Combined Cycle Power Plant, *CCPP*, considering diverse fault scenarios. The essential and more critical component in the plant self is the gas turbine, because it comprises complex dynamical subsystems which can fail due to faults in sensors, actuators and components and relies heavily on the control system affecting the reliability, availability and maintainability of the power plant. This issue motivated this research work oriented to design a diagnosis system by software for gas turbines of electric power plants. The key for a faults diagnosis system is the discrepancy between expected and actual behavior and this can be identified, on real time only if redundant information between the process variables is available (Frank, 1990). Artificial Intelligence and Control communities have developed methods to generate symptoms or signals by software, called residuals, which reflect the discrepancies in faults conditions Venkatasubramanian et al. (2003a), Venkatasubramanian et al. (2003c), Venkatasubramanian et al. (2003d). However previous to the residual generators design it is necessary and essential to determine which data requirements are required to solve a specific fault diagnosis issue.

To analyze under which conditions faults in sensors and actuators of a *GT* can be detected and isolated, the structural properties of the model are used here. The redundancy of the structure is studied using graph tools for the subsystems of the *GT* considering the available measurements. A non-linear complex dynamic model of the *GT* given by 37 algebraic and differential equations is considered to identify the required redundancy degrees for diverse fault scenarios of the units without numerical values. As result of the generic analysis, 10 relations are obtained which allow to detect faults in all components of the gas turbine unit. The rotors mechanical coupling to gas turbine unit for one side and the electric generator unit for the other side, is identified as a subsystem in which faults are undetectable and then, a diagnosis system for this subsystem is not feasible. This means, the standard instrumentation of the *GT* restricts its performance from safety and integrity point of view. On the base of this result and using the redundant graph concept (Verde & Mina, 2008), it is suggested here to add a sensor to increase the redundance and consequently to improve the fault detectability of the turbogenerator in the presence of mechanical and sensors faults. This is the main contribution of the work. The implementation of redundant graphs with specific simulated data of a *GT* validates this statement.

The work is organized as follows. The first part of Section 2 presents the philosophy behind an active supervision system by software. The second one introduces the structural framework to detect and isolate faults in a complex dynamic plant where the concept of redundancy graph is introduced. Section 3 describes shortly the model structure of the *GT* and the interconnection between units used in the study. Section 4 discusses in detail the analysis of the *GT* by graph tools considering the feasible fault scenarios in sensor and actuators. These scenarios determine the generic redundancy equations which have to be implemented in the supervision system of the *GT*. Following this analysis, the main contribution is given in first part of Section 5. Based on the Redundant Graph $\mathcal{RG}$, the subsystem without redundancy is here described and how to look for a new available variable in the graph to improve the detection capabilities and isolability properties of the turbogenerator unit. The second part of Section 5 includes some numerical results of the implementation of the detection system, and the discussions of the results and conclusions are given in Section 6.

2. Process supervision with fault diagnosis

The automatic supervision of power plants was mainly realized in the past by limit checking of important process variables. Usually alarms are raised if the limit values are exceeded and protection systems act manually or automatic. This simple procedure generates delayed alarms without detailed diagnosis. Modern methods based of system theory made possible to develop advanced fault detection and active diagnosis systems by software (Venkatasubramanian et al., 2003b). In this framework a fault for the study case is defined as deviations of the *GT* from its normal characteristics affecting the automatic system (Isermann, 2006). To develop modern automatic supervision and diagnosis system a combination of diverse methods have been developed by the safe process community of IFAC. The principle to solve a fault diagnosis problem is the **redundancy** and consistency of data in a system (Frank et al., 1999) together with model behavior in normal and fault condition. As example, for the model

$$y = 3u + 6u^2 - 4uy \qquad (1)$$

assuming that both variables (u,y) are known, one can estimate by software y, called $\hat{y}$, based on u and Eq. (1). Then, the data of y gives a redundant information and one can check the system behavior looking for the dissimilarity between $\hat{y}$ and y.

For large scale systems, a model is not so simple as the above case. Therefore, the control theory, signals processing and artificial intelligence communities have proposed diverse methodologies to supervise the system behavior. The general frameworks are described in the books (Korbicz et al., 2004), (Ding, 2008). The analytical formulation of a discrepancy assumes the existence of two or more ways, to determine variables of a process, where one way uses a mathematical model in analytical form (Blanke et al., 2003). This means, given a vector k_i integrated by a subset of known signals $\mathcal{K}_i$ of a process, any expression of the form

$$ARR(\mathcal{K}_i) = RR(k_i, \dot{k}_i, \ddot{k}_i \ldots) = 0 \qquad (2)$$

obtained from an analytical model is called an **Analytical Redundancy Relation** *ARR* for a set of detectable faults $\mathcal{F}$, if for all $\mathcal{K}_i$ consistent with the process free of faults, *ARR* is zero; and if a fault $f_i \in \mathcal{F}$ occurs, *ARR* is inconsistent or different from zero at least in a time

interval. The relations of class (2) can be obtained by different methodologies: analytical expressions, historical data, signal processing, etc. (Korbicz et al., 2004). The number of feasible *ARR*s depends strongly on the measurements signals available and the sensors location and the structure of the plant under supervision. Thus, the more variables are measured, the better performance could have the active diagnosis system.

For the GT of a combined cycle power plant, the next issues are formulated here:

- Which are the technical conditions to get a complete fault detection? and
- How could one guarantee full scope faults isolability?

The study and considerations to solve the first task are the main contributions of this work, while the second task is focused on the selection of ARRs for specific fault sets. In particular, one should select a method which captures all possible solutions of a diagnosis issue taking in account the available measurements. Since, to deal with large scale complex dynamic systems, the generic structural approaches are more appropriated than the numerical methods; one may select analysis tools based on structure properties, which have been proposed to achieve this goal, as Structural Analysis (Cassal et al., 1994), Geometric Approach (De-Persis & Isidori, 2001), Linear Structured Systems (Dion et al., 2003), or Bond Graph (Mukherjee et al., 2006). In particular, Structural Analysis (SA) framework, which is based on graph theory, allow to study the system capabilities to detect and isolate faults. This framework has two relevant characteristics: allows dealing with complex and large scale systems and does not require numeric parameters information. So, with this approach, to know if there are redundant variables in a system, only its structure without explicit numerical values plays an important role. This is the main reason to select SA for the GT fault issues in this work.

2.1 System description by a graph

The Structural Analysis is based on relationships between variables given in the form of a bipartite graph or equivalently as an boolean incidence matrix, and it can be used in the early design phase of a supervisory system. Here one describes briefly the main tools of the *SA*, including the redundancy graph concept as an extension of the analytical redundancy relation used in the model-based fault diagnosis methods (Verde & Mina, 2008).

A system can be described by a bipartite graph $\mathcal{G}$ with its variables $\mathcal{V}$ and its equations $\mathcal{C}$ as node sets where there are edges connecting constraint with variables (Gross & Yellen, 2006). There are two form to shows the connection between nodes in a graph, by a diagram or a binary incidence matrix. The following description formalizes this concept.

Definition 1: Let a dynamic system be given by

$$\dot{x} = f_m(x, \tilde{x}, u, \theta, f, \overline{f}), \quad x \epsilon \mathfrak{R}^n, u \epsilon \mathfrak{R}^{n_u}, f \epsilon \mathfrak{R}^f, \overline{f} \epsilon \mathfrak{R}^d \tag{3}$$

$$y = h(x, \tilde{x}, u, \theta, f, \overline{f}), \quad y \epsilon \mathfrak{R}^{n_y} \tag{4}$$

$$0_p = m(x, \tilde{x}, u, \theta, f, \overline{f}), \quad \tilde{x} \epsilon \mathfrak{R}^s, 0_p \epsilon \mathfrak{R}^p \tag{5}$$

with u and y known variables, θ the parameter vector, f and $\overline{f}$ faults to be detected and neglected respectively. The bipartite graph associated to Equations (3, 4, 5) is defined by the graph $\mathcal{G} = (\mathcal{C} \cup \mathcal{V}, \mathcal{E})$ where the edges connecting set $\mathcal{E}$ is given by

$$e_{ij} = \begin{cases} (c_i, v_j) & \text{if and only if } v_j \text{ appers in } c_i \\ 0 & \text{on the contrary} \end{cases}$$

Using the matrix description an edge e_{ij} is given by • in row i, column j.

According to Eqs. (3,4,5), in the graph description, the constraints nodes set C has cardinality $|C| = 2n + n_y + p$ and the variables nodes set $V = X_g \cup K \cup F \cup \bar{F}$ is defined by

- the unknown variables set $X_g = X \cup \dot{X} \cup \tilde{X}$ with cardinality $2n + s$;
- the known variables set $K = U \cup Y$; where the exogenous set U and the measurements set Y have cardinality n_u and n_y respectively and then $n_k = |K| = n_u + n_y$;
- the fault and disturbance (neglected faults) sets F and $\bar{F}$ have cardinality f and d respectively;
- Each state variable x_i involves a constraint

$$\dot{x}_i = \frac{dx_i}{dt} \tag{d}$$

Since a fault f which changes the normal behavior of constraint c_i means that the edges e_{ij} for any j are sensitive to f, the graph description $G(C \cup V, \mathcal{E})$ allows to consider faults indifferently, as changes in the subset C or as an input node subset F without numerical values. This is an advantage to study the system diagnosis capability.

Example.

To show the simplicity to get a bipartite graph and the matching assignment, consider the following simple differential algebraic system

$$\dot{x}_1 = g_1(x_1, u) \tag{c1}$$

$$x_2 = g_2(x_1) \tag{c2}$$

$$y = g_3(x_2) \tag{c3}$$

In this simple case,

- the variables nodes set is given $V = \{x_1, x_2, u, y, \dot{x}_1\}$,
- the constraints nodes set $C = \{c1, c2, c3, d\}$ where d corresponds to the constraint $\dot{x}_1 = \frac{dx_1}{dt}$

The bipartite graph is given in Fig. 1 where a shadowed circle denotes a constraint. The respective description incidence matrix, *IM*, is shown in Table 1.

To establish relations between the variables of V and the constraints of C, the edges of $\mathcal{E}$ has to be oriented. This is equivalent to define paths joining nodes of V with nodes of C. This process in which each node c_i is used to express only a node of V is called **matching process**. In the incidence matrix framework, the matching process means rows and columns permutations, in which the symbol $\oplus$ in the row i and column j denotes that the constraint node i is used to get the variable node j. The symbols (• →) denotes initial node. There are diverse matching algorithms (Krysander et al., 2008), however not all can be used for fault detection analysis, since the redundant relation between variables and the causality of a process have to be considered. From fault detection point of view, only matchings with redundant information are relevant. The Structural Analysis, *SA* deals with the systematic procedures to get the redundant relations from a bipartite graph without numerical value (Blanke et al., 2003).
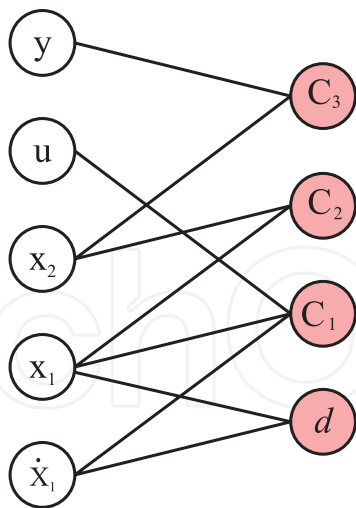
Fig. 1. Bipartite graph of system (c1,c2,c3,d)

\	x_1	x_2	$\dot{x}_1$	u	y
$c1$	•		•	•	
$c2$	•	•			
$c3$		•			•
d	•		•		

Table 1. Incidence Matrix of the Bipartite Graph

From matrix of Table 1, the matching with initial node y shown in Table 2 is generated and the path is read as follow: using the constraints $c3$ together with the variable y, one matches x_2, as second step, using x_2 with $c2$, one reaches x_1 and later one, by d the unknown variable $\dot{x}_1$ is determined and finally the goal node u can be evaluated by $c1$, since the rest of variables has been before calculated. Therefore if the variables pair (u,y) are known, one can estimate $\hat{u}$ by data of y and the path of evaluation, i.e.

$$\hat{u} = g_1^{-1}(d(g_2^{-1}(g_3^{-1}(y)))) \tag{6}$$

Thus, a real time comparison of $\hat{u}$ with the data of u detects any abnormal conditions involved in any of the constraints set ($c1$, $c2$, $c3$) and the difference

$$r(t) = \hat{u} - u = g_1 \circ d \circ g_2 \circ g_3(y) - u \tag{7}$$

is a symptom signal, where $\circ$ denotes concatenation. The evaluation of (7), called residual signal $r(t)$, is zero in normal ideal condition and different from zero in abnormal conditions; since $r(t)$t only depends of the data pair (u,y), then it is a analytical redundant relation, *ARR*.

\	y	x_2	x_1	$\dot{x}_1$	u
$c3$	• →	⊕			
$c2$		•	⊕		
d			•	⊕	
$c1$			•	•	→ ⊕

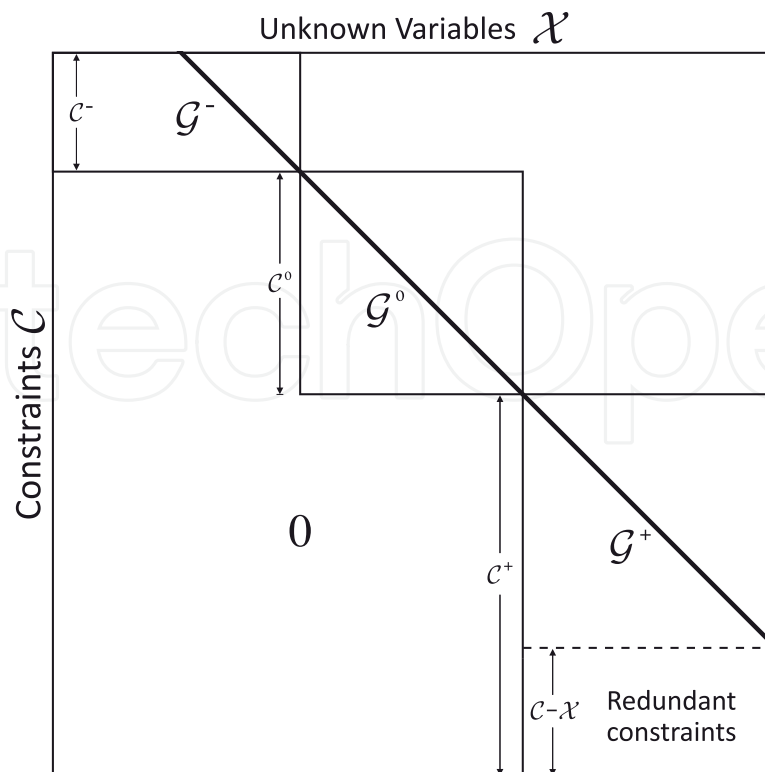Table 2. Path to evaluate u by y

Fig. 2. Canonic Decomposition of a generic Incidence Matrix

2.2 Redundancy in a graph

In the framework of Structural Analysis, the existence of *ARR*s or relations of class (2), implies that the graph has more constraints than unknown variables and the maximum number of redundancy relations is bounded by $|\mathcal{C}| - |\mathcal{X}_g|$ (Krysander et al., 2008). Thus, the starting point of the faults structural analysis is the canonical Dulmage-Mendelsohn decomposition of the graph in three sub-graphs: the over-constrained $\mathcal{G}^+$ with more constraints than unknown variables $\mathcal{X}_g^+$, the just-constrained $\mathcal{G}^0$ with the same number of constraints and unknown variables, and the under-constrained $\mathcal{G}^-$ with less constraints than unknown variables. Fig. 2 shows the generic structure of a decomposed graph in three subgraphs. Furthermore, faults which affect constraints involved in $\mathcal{G}^0$ and $\mathcal{G}^-$ are not detectable (Blanke et al., 2003).

The relations between variables set $\mathcal{K}^+$ with constraints $\mathcal{C}^+$ in which the set $\mathcal{X}_g^+$ has been substituted, determine the *ARR*'s. This is equivalent to give an orientation to each edge, eliminating the set $\mathcal{X}_g^+$ using some members of $\mathcal{C}^+$. Once a matching is obtained in $\mathcal{G}^+$, the involved constraints can be interpreted as operators from a set of known variables to others where the path is determined by a concatenation process following an oriented graph. The concatenation algorithm for linear constraints is reduced to the Mason's method (Mason, 1956) and there are diverse ways to select the *ARR*s from the paths of a graph.

For the particular graph $\mathcal{G}$ given in Fig. 1, the Dulmage-Medelsohn decomposition identifies the condition $\mathcal{G}^+ = \mathcal{G}$ with 4 constraints, 3 unknown variables and the pair (u,y) as known nodes. Since the paths from y to u o viceversa pass by all constraints ($c3$, $c2$, d, $c1$), all faults associated to sensors, actuator and constraints are generic detectable. Thus, the path of Table

2 is a base to generate the *ARR*s for the model (c1,c2,c3), Eq. 7 is a particular analytical redundant relation.

2.3 Redundant graph definition

Let

$$\mathcal{K}_i = \mathcal{U}_{si} \cup y_i \tag{8}$$

be a subset of known variables matched with the subset of constraints $\mathcal{C}_i$, initial vertex of $\mathcal{U}_{si}$ and target vertex y_i, then

$$\mathcal{RG}_i(\mathcal{C}_i; \mathcal{U}_{si}; y_i) \tag{9}$$

is a **Redundant Graph** if

- Paths between the vertices of $\mathcal{U}_{si}$ and the target y_i are consistent and they can be obtained concatenating $\mathcal{C}_i$ without faults, and
- At fault condition, there is a lack of consistency in some paths between $\mathcal{U}_{si}$ and Y_i for any elements of the constraints set.

In the matrix framework, symbols $\bullet \rightarrow$ and $\rightarrow \bullet$ are used for initial and target vertices respectively. Note that for an specific $\mathcal{RG}$, members of $\mathcal{U}_{si}$ are independent variables which are correlated with y_i by paths of the redundant graph. In this framework, faults which are unknown a priori are considered inconsistent vertices in the graph.

From Table 2, one identifies the path from *y* to *u* as a $\mathcal{RG}(\{c3, c2, d, c1\}; y; u)$; with *y* as initial node and *u* as target node. Other graph can be built if *u* is the inicial node and *y* the target node. Since both paths pass by the same nodes, then both are equivalent sets from redundant point of view.

General advantages of the $\mathcal{RG}_i$ are:

- One can generated distributed subgraphs where cause and effect can be indistinctly handled;
- One can build the faults symptoms (faults signature) from the $\mathcal{RG}_i$, without numeric values of a system model. This is useful to search new sensors which improve the faults signature.
- For large scale systems, the redundant subgraph allows the determination of correlated variables without numerical values. This has been used to isolate faults Mina et al. (2008).

2.3.1 $\mathcal{RG}$ algorithm

The following algorithm summarize the steps to get redundant graphs $\mathcal{RG}$s assuming known the bipartite graph of the system (3, 4, 5)

Step 1. Calculate the canonical decomposition of $\mathcal{G}$ using only the unknown variables set $\mathcal{X}$ (Pothen & Fan, 1990).

Step 2. Identify the subgraph $\mathcal{G}^+$.

Step 3. Eliminate the constraints set $\mathcal{C}_e$ which involves not invertible functions and build $\mathcal{C}_{inv}^+ = \mathcal{C}^+ \setminus \mathcal{C}_e$.

Step 4. Calculate the possible maximal number of redundant graphs given by

$$Max_{rr} = \left| \mathcal{C}_{inv}^{+} \right| - \left| \mathcal{X}^{+} \right|$$

Step 5. Initialize the number of initial node $n_i = 1$ in the search and the number of assigned redundant graph $n_{\mathcal{GR}} = 0$.

Step 6. Calculate the possible distinct combinations of the initial nodes for each target, selecting n_i nodes out of $n_k - 1$, with n_k the cardinality of set $\mathcal{K}$; this means

$$I = {}^{n_k - 1}\mathbf{C}_{n_i} = \binom{n_k - 1}{n_i} = \frac{(n_k - 1)!}{(n_k - 1 - n_i)!(n_i)!} \quad \text{for each target node}$$

Step 7. Assign the orientations of the I graphs using the set $\mathcal{C}_{inv}^{+}$ for each target node including the cycle graphs (no diagonal submatrix) and constraints of the class d.

Step 8. Bring up the number $n_{\mathcal{GR}}$ according the assigned redundant graphs; if $n_{\mathcal{GR}} = Max_{rr}$, end the algorithm, otherwise continue.

Step 9. If $n_i = n_k - 1$, end the algorithm, on the contrary $n_i = n_i + 1$ an return to step 6.

3. Gas turbine description

The *GT* behavior model used at this work simulates electrical power generation in a combined cycle power plant configuration with two *GT*, two heat recovery-steam generators and a steam turbine. At ISO conditions, the ideal power delivered for each *GT* generates 80*MW* and the steam turbine 100*MW*. This model may go from cold startup to base load generation. The main components of the *GT* shown in Fig. 3 are: compressor *C*, combustion chamber *CC*, gas turbine section *T*, electric generator *EG*, and heat recovery *HRSG*.
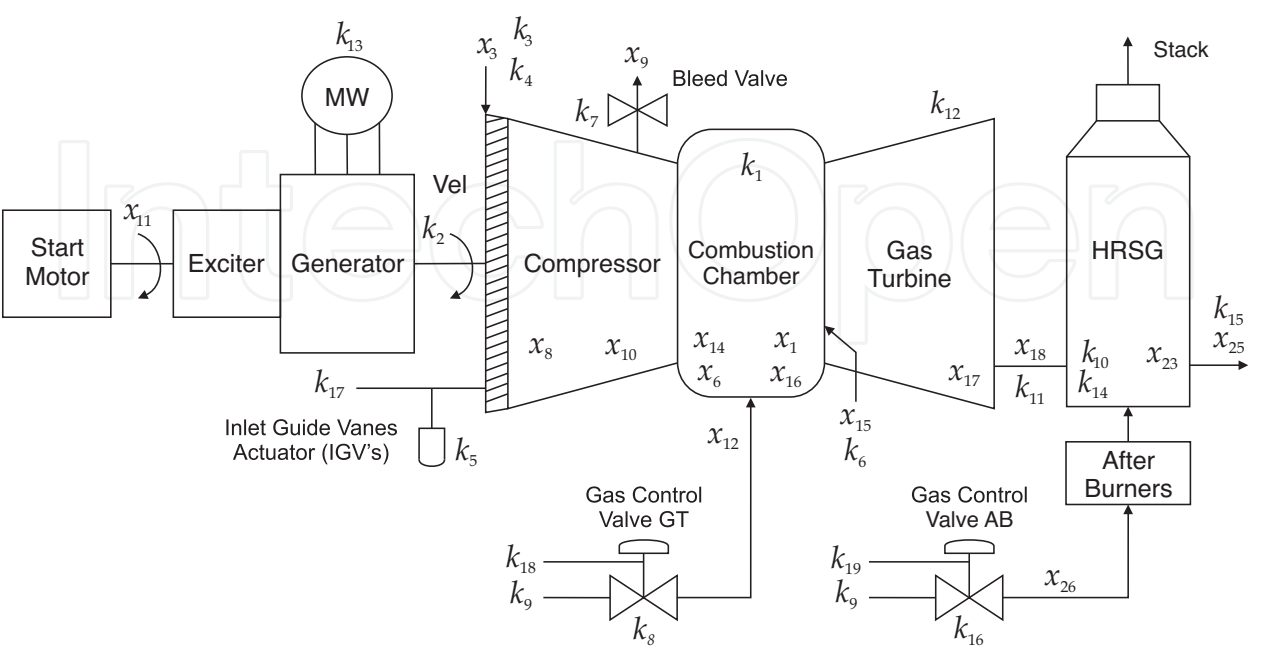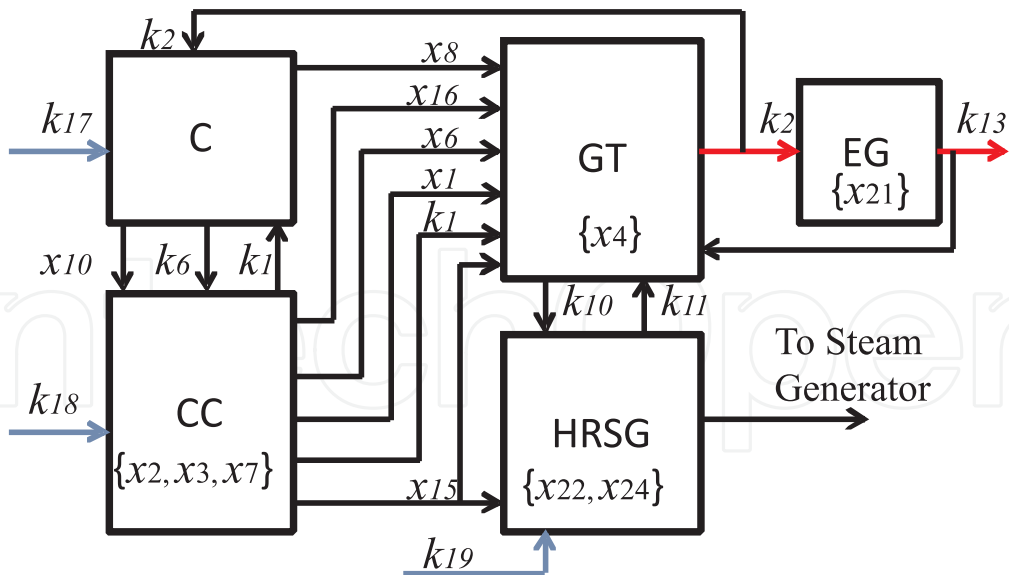

Fig. 3. Components of the Gas Turbine

Fig. 4. Gas Turbine Variables Interconnection

The *GT* unit has two gas fuel control valves; the first supplies gas fuel to CC, and the second one supplies gas fuel to heat-recovery afterburners (starting a second- additional combustion at heat recovery for increasing the exhaust gases temperature). A generic compressor bleed valve extracts air from compressor during *GT* acceleration, avoiding an stall or surge phenomena. Also the *GT* unit has an actuator for the compressor inlet guide vanes, *IGVs*, to get the required air flow to the combustion chamber. The dynamic nonlinear model is developed in (Delgadillo & Fuentes, 1996) and it is integrated by n_c = 28 constraints, n_s = 19 static algebraic constraints, and n = 9 dynamic-differential constraints. Concerning the variables one can identify 27 unknown variables x_i and 19 known variables k_i. The generic architecture and interconnection of the GT's components are described by the block scheme given in Fig. 4. The variables and parameters for each block of the scheme are related by the constraints described in table 3. The variables are given in Appendix 8 and the description of the functions and parameters can be consulted in (Sánchez-Parra et al., 2010).

4. Analysis of the structure for the gas turbine

Considering constraints and variables of the model described in Table (3) the following sets for the graph description are identified:

- The set of known variables is given by

$$\mathcal{K} = \mathcal{Y}_s \cup \mathcal{Y}_a \cup \mathcal{U}_p \cup \mathcal{U}_c \qquad (10)$$

with cardinality 19. The process sensors is determined by the set

$$\mathcal{Y}_s = \left\{ k_1, k_2, k_6, k_{10}, k_{11}, k_{12}, k_{13}, k_{14}, k_{15} \right\} \qquad (11)$$

with $|\mathcal{Y}_s|$ =9; the position transducers from actuators define the set $\mathcal{Y}_a$ ={k_5, k_7, k_8, k_{16}}; the external physical variables determine the set $\mathcal{U}_p$ = {k_3, k_4, k_9}; and the control signals defines the set $\mathcal{U}_c$ = {k_{17}, k_{18}, k_{19}}.

- There are 28 physical parameters θ_i which are assumed constant in normal conditions Sánchez-Parra & Verde (2006).

Compressor Unit, C	Combustion Chamber Unit, CC
c1: $0 = f(x_1, x_6, k_1, \theta_0)$ c2: $0 = f(x_3, k_1, k_2, k_3, k_5, \theta_1, \theta_2, \theta_3)$ c3: $0 = f(x_3, x_8, k_1, k_3, \theta_4, \theta_5)$ c4: $0 = f(k_1, k_3, k_4, k_6, \theta_5)$ c5: $0 = f(x_9, k_1, k_3, k_6, k_7, \theta_6)$ c6: $0 = f(x_3, x_9, x_{10})$ c7: $0 = f(x_5, k_5, k_{17}, \theta_{25})$ d1: $0 = x_5 - \frac{dk_5}{dt}$	c8: $0 = f(x_6, x_{12}, k_1, k_8, k_9, \theta_7)$ c9: $0 = f(x_{10}, x_{12}, x_{14})$ c10: $0 = f(x_6, x_{15}, k_1, \theta_{21})$ c11: $0 = f(x_1, x_2, x_{14}, x_{15}, \theta_{17})$ d2: $0 = x_2 - \frac{dx_1}{dt}$ c12: $0 = f(x_1, x_6, x_7, x_{10}, x_{12}, x_{14}, k_6, \theta_8,$ $\theta_9, \theta_{17}, \theta_{18}, \theta_{19})$ d3: $0 = x_7 - \frac{dx_6}{dt}$ c13: $0 = f(x_{13}, k_8, k_{18}, \theta_{26})$ d4: $0 = x_{13} - \frac{dk_8}{dt}$
Gas Turbine Unit, GT	**Heat Recovery Unit, HR**

c14: $0 = f(x_{10}, x_{12}, x_{16}, k_6, \theta_8, \theta_9, \theta_{18})$
 c15: $0 = f(x_1, x_{17}, k_1, k_{10}, \theta_{10})$
 c16: $0 = f(x_1, x_{16}, x_{17}, x_{18}, k_1, k_{10})$
 c17: $0 = f(x_6, k_{11}, k_{12})$
 c18: $0 = f(x_6, k_1, k_{10}, k_{11}, \theta_{10})$
 c19: $0 = f(x_{19}, k_2, \theta_{11})$
 d5: $0 = x_4 - \frac{dk_2}{dt}$
 c20: $0 = f(x_4, x_8, x_{11}, x_{15}, x_{16}, x_{18},$
           $x_{19}, k_2, k_{13}, \theta_{20})$
	c23: $0 = f(x_{23}, k_{10}, k_{14}, \theta_0)$
	 c24: $0 = f(x_{25}, k_3, k_{10}, k_{15}, \theta_{23})$
	 c25: $0 = f(x_{26}, k_9, k_{10}, k_{14}, k_{16}, \theta_{24})$
	 c26: $0 = f(x_{15}, x_{23}, x_{24}, x_{25}, x_{26}, \theta_{16})$
	 d7: $0 = x_{24} - \frac{dx_{23}}{dt}$
	 c27: $0 = f(x_{15}, x_{22}, x_{23}, x_{26}, k_{11}, k_{14}, \theta_9,$
	           $\theta_{16}, \theta_{18}, \theta_{19})$
	 c28: $0 = f(x_{27}, k_{16}, k_{19}, \theta_{27})$
	 d8: $0 = x_{22} - \frac{dk_{14}}{dt}$
	 d9: $0 = x_{27} - \frac{dk_{16}}{dt}$

Electric Generator Unit EG	
c21: $0 = f(x_{20}, x_{21}, k_{13}, \theta_{12}, \theta_{13}, \theta_{14}, \theta_{15})$ c22: $0 = f(x_{20}, x_{21}, k_2, \theta_{22})$ d6: $0 = x_{21} - \frac{dx_{20}}{dt}$	

Table 3. *GT* Model Equations with the variables meaning given in the appendix

- The constraints set is given by 19 static constraints and 9 state constraints which require their additional constraints (di) and known variables. Then the constraints set has cardinality 37 and is given by

$$\mathcal{C} = \{c1, c2, \ldots, c28\} \cup \{d1, d2, \ldots, d9\} \tag{12}$$

- The unknown variables are 27 and define the set

$$\mathcal{X} = \tilde{\mathcal{X}} \cup \mathcal{X} \cup \dot{\mathcal{X}} \tag{13}$$

where the dynamic unknown variables set has cardinality 4 and is given by

$$\mathcal{X} = \{x_1, x_6, x_{20}, x_{23}\}, \tag{14}$$

the unknown variables set which are related by static relations of cardinality $|\tilde{\mathcal{X}}| = 14$ are

$$\tilde{\mathcal{X}} = \left\{ x_3, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{14}, x_{15}, x_{16}, x_{17}, x_{18}, x_{19}, x_{25}, x_{26} \right\} \qquad (15)$$

and the differential of the state variables are

$$\dot{\mathcal{X}} = \left\{ x_5, x_2, x_7, x_{13}, x_4, x_{21}, x_{24}, x_{22}, x_{27} \right\} \qquad (16)$$

with $| \dot{\mathcal{X}} | = 9$.

Considering the above described sets of variables and constraints, the Incidence Matrix, IM, of dimension (37×27) is first obtained and this is the start point of the structural analysis. Using Matlab (MATLAB R2008, 2008) the decomposed incidence matrix given in Fig. 5 is obtained. The bottom sub-matrix $IM^+ \in \mathbb{I}^{30 \times 20}$ is associated to $\mathcal{G}^+$ and $IM^0 \in \mathbb{I}^{7 \times 7}$ for $\mathcal{G}^0$ with $\mathcal{G}^- = \varnothing$. The diagnosticability analysis of the first part of the analysis takes into account only the over-constrained $\mathcal{G}^+$. The issue of the undetectability of the subgraph $\mathcal{G}^0$ will be addressed in Section 5.

4.1 Redundancy of the *GT* structure

Based on the subgraph $\mathcal{G}^+$, the maximum number of $\mathcal{RG}$ is given by $|\mathcal{C}^+| - |\mathcal{X}^+| = 10$. Considering the matching sequences described in the first 20 rows of Fig. 6 and concatenating these with other 10 constraints, Table 4 is obtained and the failed components which can be detected in the *GT* are identified. The third column indicates the variables used to detect faults involved in the respective set of constraints for each $\mathcal{RG}$. One can see that some faults can be supervised using two $\mathcal{RG}$s. As example faults in the component of constraint c_9 can be supervised by the graph $\mathcal{RG}_7$ or $\mathcal{RG}_8$ with different subsets of $\mathcal{K}$.

Table 4 is obtained and the failed components which can be detected in the *GT* are identified.

5. Diagnosticability improvement in the *GT*

The subsystem $\mathcal{G}^0$ given at the top of the matrix in Fig. 5 describes the process without redundant data and and the unique matched graph is shown in Fig. 7. It involves some of turbogenerator variables given in Table 3. Without redundant relations, it is impossible to detect a fault at the turbogenerator section with the assumed instrumentation. Giampaolo (2003) calls this subsystem, *GT* Thermodynamic Gas and includes the non-measured variables: compressor energy and rotor-friction energy (x_8, x_{19}); exhaust gases enthalpy and combustion chamber gases enthalpy (x_{18}, x_{16}); exhaust gases density x_{17}, rotor acceleration x_4 and the start motor power x_{11}. Thus, the main concern of this section is the identification of the unknown variables, which can be measured and converted to new known variables. So, with this the graph decomposition $\mathcal{G}^0$ will be empty and the getting of the respective *ARR* yields by the new measurement.

5.1 Graph structure modification

The oriented graph of $\mathcal{G}^0$ assuming the known variables subset $\mathcal{K}$ is shown in Fig. 7. The absence of paths which link a subset of known variables is recognized. The unknown variables $\mathcal{X}^0$ cannot be bypassed in any path and as consequence does not exist a $\mathcal{RG}$.
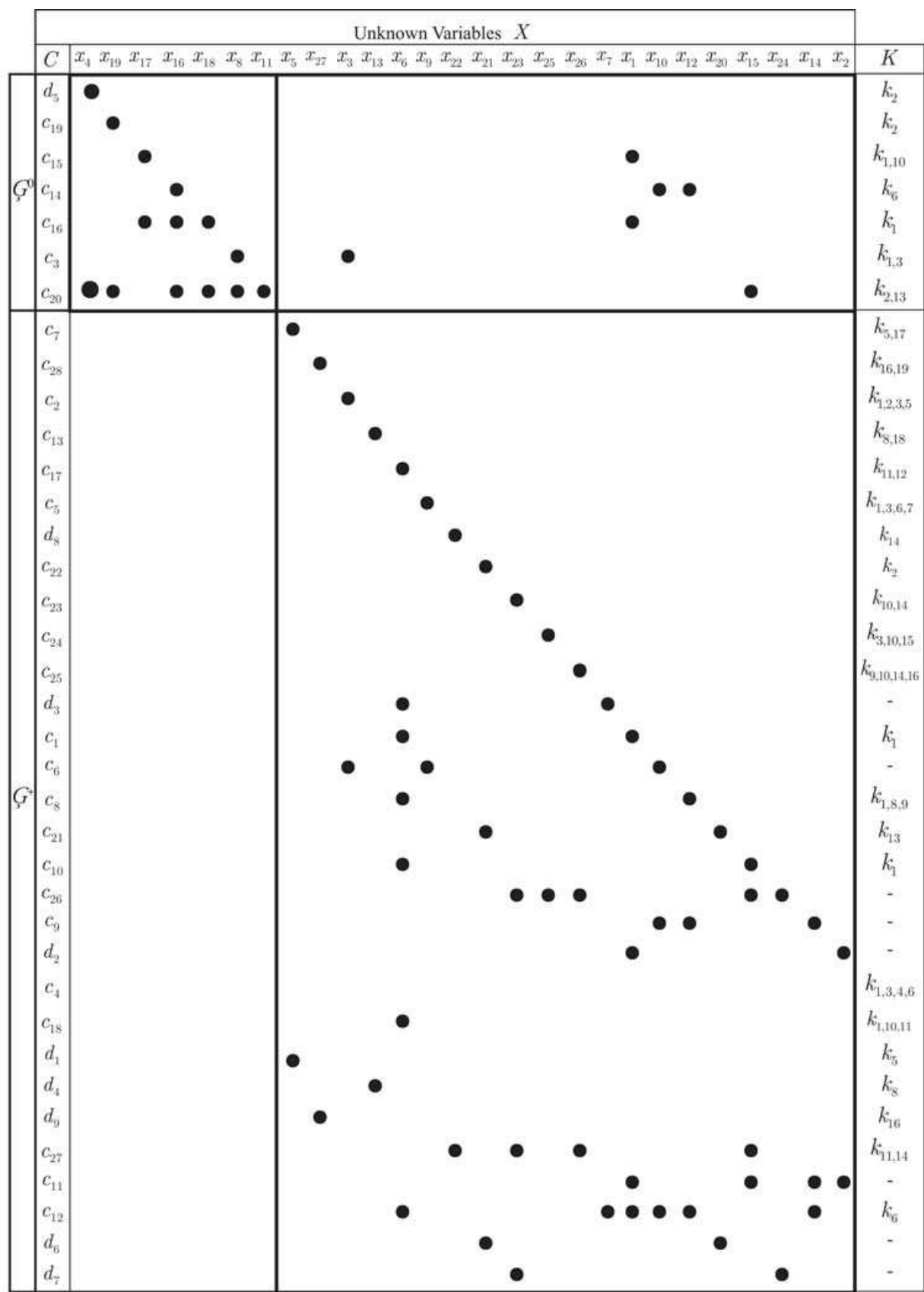
Fig. 5. Decomposed Incidence Matrix for the GT, where G^0 and G^+ are identified by blocks

$\mathcal{RG}$'s	Used Constraints $\mathcal{C}^+$	Known variables $\mathcal{K}$
$\mathcal{RG}_1$	c_4	k_1, k_3, k_4, k_6
$\mathcal{RG}_2$	c_{17}, c_{18}	$k_1, k_{10}, k_{11}, k_{12}$
$\mathcal{RG}_3$	d_1, c_7	k_5, k_7
$\mathcal{RG}_4$	d_4, c_{13}	k_8, k_{18}
$\mathcal{RG}_5$	d_9, c_{28}	k_{16}, k_{19}
$\mathcal{RG}_6$	$d_8, c_{10}, c_{17}, c_{23}, c_{25}, c_{27}$	$k_1, k_9, k_{10}, k_{11}, k_{12}, k_{14}, k_{16}, k_{19}$
$\mathcal{RG}_7$	$d_2, c_1, c_2, c_5, c_6, c_8, c_9, c_{10}, c_{11}, c_{17}$	$k_1, k_2, k_3, k_5, k_6, k_7, k_8, k_9, k_{11}, k_{12}$
$\mathcal{RG}_8$	$d_3, c_1, c_2, c_5, c_6, c_8, c_9, c_{12}, c_{17}$	$k_1, k_2, k_3, k_5, k_6, k_7, k_8, k_9, k_{11}, k_{12}$
$\mathcal{RG}_9$	d_6, c_{21}, c_{22}	k_2, k_{13}
$\mathcal{RG}_{10}$	$d_7, c_{10}, c_{17}, c_{23}, c_{24}, c_{25}, c_{26}$	$k_1, k_3, k_9, k_{10}, k_{11}, k_{12}, k_{14}, k_{15}, k_{16}$

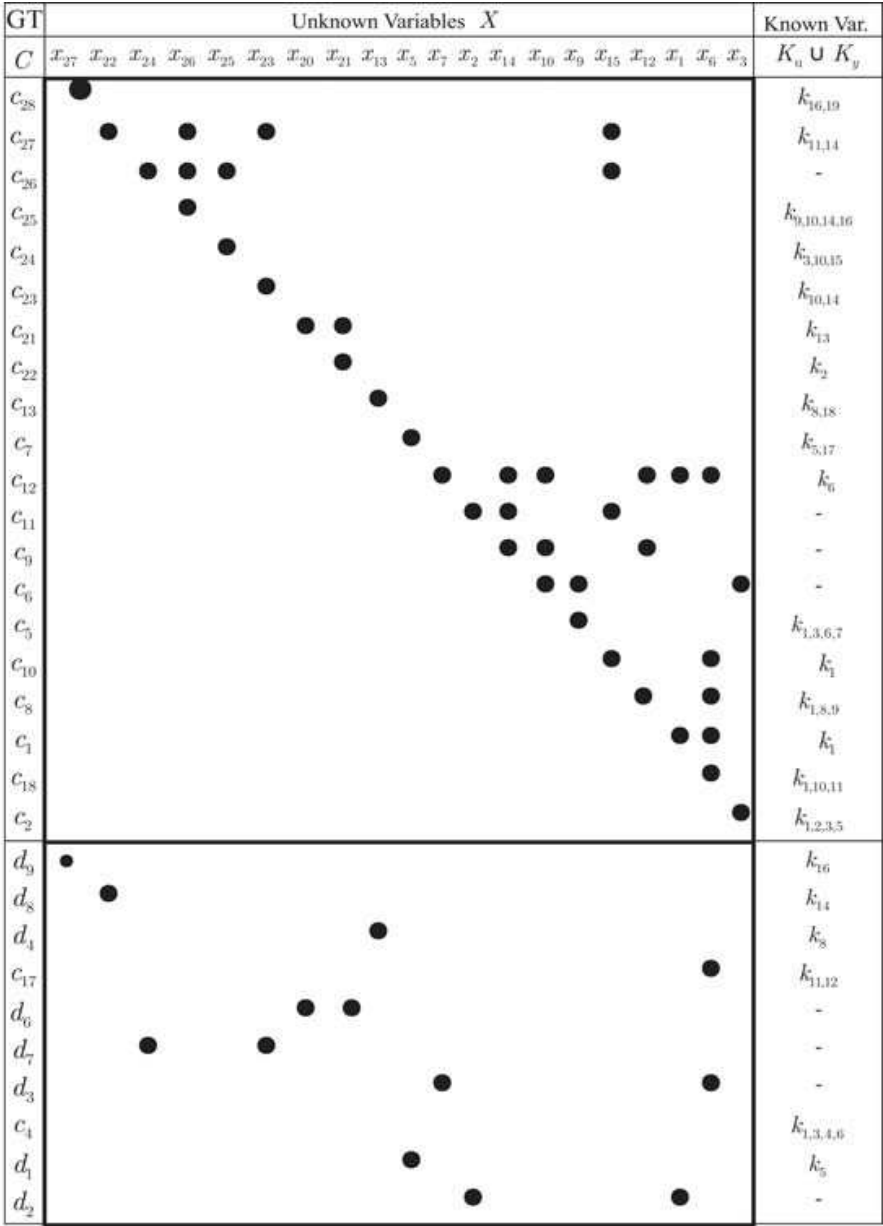Table 4. Redundant Graphs obtained from $\mathcal{G}^+$

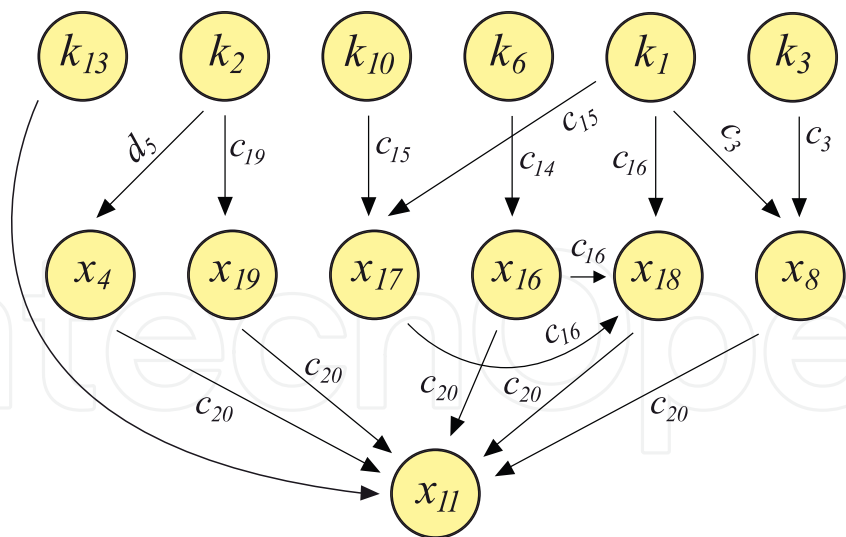Fig. 6. Matching for the GT to get $10\mathcal{GR}$

Fig. 7. Subgraph $\mathcal{G}^0$ without redundant information

To determine which variables of $\mathcal{G}^0$ could modify this lack of detectability, paths which satisfy the $\mathcal{RG}$ conditions assuming new sensors has to be builded. Then, one has to search for paths between known variables which pass by the constraint c_{20}. On the other hand, from the incidence matrix of the Table 5 one can identify that variable x_{11} appears only in the constraint c_{20}. Thus, there are not two different paths to evaluate it. To pass by c_{20} the only possibility is to asume that x_{11} is measurable.

Taking into account physical meaning of the set $\mathcal{X}^0$, it is feasible to assume that the start motor power x_{11} is known. This proposition changes the *GT* structure, transforming the whole structure to an over-constrained graph. In other words adding a dynamo-meter to the *GT* instrumentation, x_{11} became a new known variable, $k_{20} = x_{11}$, and allows the construction of the redundant graph described in Table 5. One verify that estimating first the set $\{x_1, x_3, x_{10}, x_{12}, x_{15}\}$ by subsets of $\mathcal{K}$ and $\mathcal{C}^+$, one can estimate $\hat{x}_{11}$ following the path. Thus, the relation

$$r(t) = \hat{k}_{20} - k_{20} \tag{17}$$

can be used as to generate a residual and the respective ARR_{11} depends on the variables set

$$\mathcal{K}^* = \{k_1, k_2, k_3, k_5, k_6, k_7, k_8, k_9, k_{10}, k_{11}, k_{12}, k_{13}, k_{20}\} \tag{18}$$

and the set of constraints of the turbogenerator

$$\mathcal{C}^* = \{c_1, c_2, c_3, c_5, c_6, c_8, c_{10}, c_{14}, c_{15}, c_{16}, c_{17}, c_{19}, d_5, c_{20}\} \tag{19}$$

Thus, any changes in the parameters and the functions involved in this set of constraints generates an inconsistent in the evaluation of the target node $\hat{k}_{20}$.

5.2 Simulation results
To validate the obtained redundant relation, a change in the friction parameter $\Delta\theta_{11} = 2$ in $c19$ of the turbogenerator non linear model has been simulated. The time evolution of the

$\mathcal{C}^0$	$\mathcal{X}^+$	$\mathcal{K}$	x_4	x_{19}	x_{17}	x_{16}	x_{18}	x_8	$k_{20}=x_{11}$
d5		$k_2\to$	⊕						
c19		$k_2\to$		⊕					
c15	x_1	$k_1,k_{10}\to$			⊕				
c14	x_{10},x_{12}	$k_6\to$				⊕			
c16	x_1	$k_1\to$			•	•	⊕		
c3	x_3	$k_1,k_3\to$						⊕	
c20	x_{15}	$k_2,k_{13}\to$	•	•		•	•	•	$\to$⊕

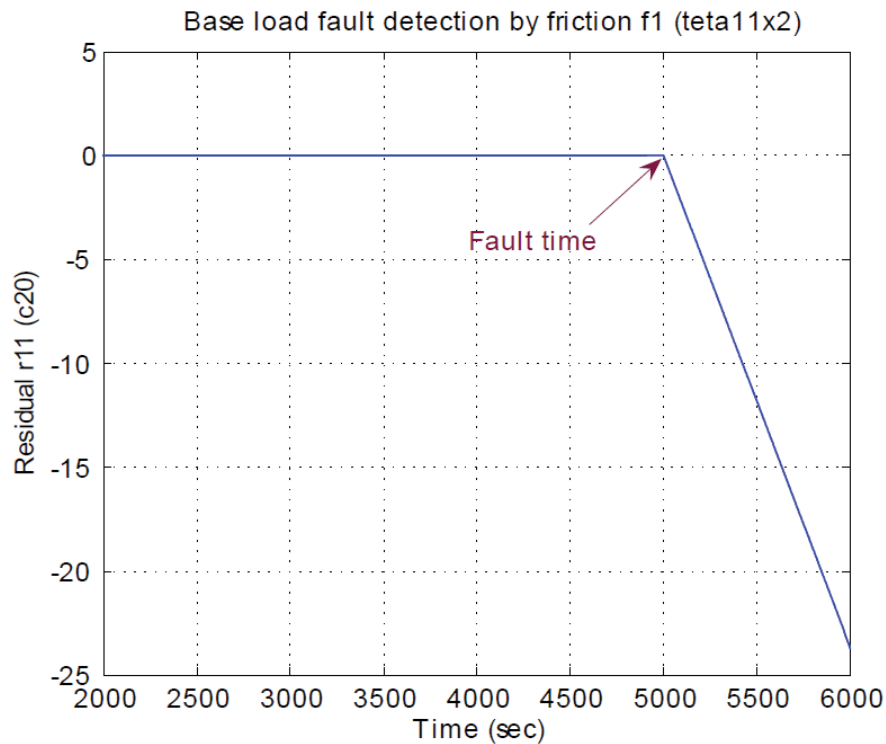Table 5. Matching Sequence of $\mathcal{G}^0$ to get Fault Detectability

Fig. 8. Residual generated by the new ARR_{11} detecting friction fault at $5000s$

residual (17) for a fault appearing at $5000s$ is shown in Fig. 8. The fast response validates the detection system. Note that during the analysis of the detection issue, any numerical value of the turbine model can be used, giving generality to this result. The values set is used for the implementation of the residual or ARR, but not in the analysis.

6. Conclusions

A fault detection analysis is presented focused on redundant information of a gas turbine in a *CCCP* model. The study using the structural analysis allows to determine the *GT's* monitoring and detection capacities with conventional sensors. From this analysis it is concluded the existence of a non-detectable fault subsystem. To eliminate such subsystem, a

reasonable proposition is the measure of the *GT*'s start motor power. Considering the new set of known variables and using the structural analysis, eleven *GT*'s redundant relations or symptoms generation are obtained. From these relations one identified that a diagnosis system can be designed for faults in sensors, actuators and turbo-generator. Since all constraints are involved at least one time in the 10 $\mathcal{RG}$s of Table 4 or in Eq. (17). This means, a diagnosis system could be designed integrating the residuals generator with a fault isolation logic which has to classify the faults. Due to space limitation it is reported here results only for a mechanical fault in the friction parameter. Using the eleven $\mathcal{RG}$ obtained here, one can achieve a whole fault diagnosis for any set of parameters.

7. Acknowledgement

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9. Appendix

k_1	Compressor discharge pressure	x_5	Compressor IGV position rate
k_2	Turbogenerador rotor speed	x_6	CC gas temperature
k_3	Atmospheric pressure	x_7	CC gas rate temperature
k_4	Outlet temperature	x_8	Compressor energy
k_5	Compressor IGV position	x_9	Compressor bleed air flow
k_6	Compressor air discharge temperature	x_{10}	Compressor outlet air flow
k_7	Compressor air bleed valve position	x_{11}	Starting motor power
k_8	Gas turbine fuel gas valve position	x_{12}	CC gas fuel flow
k_9	Inlet fuel gas valves pressure	x_{13}	GT fuel gas valve position rate

k_{10}	Heat recovery pressure	x_{14}	CC inlet gas flow
k_{11}	Exhaust gas temperature	x_{15}	CC outlet gas flow
k_{12}	Blade path temperature (BPT)	x_{16}	CC gas enthalpy
k_{13}	Electrical generator power output	x_{17}	GT exhaust gas density
k_{14}	Heat recovery gas temperature	x_{18}	GT exhaust gas enthalpy
k_{15}	Heat recovery gas outlet temperature	x_{19}	GT energy friction losses
k_{16}	Afterburner fuel gas valve position	x_{20}	Electrical generator power angle
k_{17}	IGV control signal	x_{21}	Electrical generator power rate angle
k_{18}	GT fuel gas valve control signal	x_{22}	Heat recovery gas rate temperature
k_{19}	AB fuel gas valve control signal	x_{23}	Heat recovery gas density
k_{20}	Starting motor power	x_{24}	Heat recovery gas rate density
x_1	CC gas density	x_{25}	Heat recovery outlet gas flow
x_2	CC gas rate density	x_{26}	AB gas fuel flow
x_3	Compressor inlet air flow	x_{27}	AB fuel gas valve position rate
x_4	Turbogenerator rotor speed rate	θ_4	Compressor air density
θ_{11}	GT rotor friction parameter	θ_{20}	GT rotor inertia

Table 6. Variables and Parameter Definition of the Gas Turbine Model

Gas Turbines

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This book is intended to provide valuable information for the analysis and design of various gas turbine engines for different applications. The target audience for this book is design, maintenance, materials, aerospace and mechanical engineers. The design and maintenance engineers in the gas turbine and aircraft industry will benefit immensely from the integration and system discussions in the book. The chapters are of high relevance and interest to manufacturers, researchers and academicians as well.

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