16

Phase Dynamics of Superconducting Junctions under Microwave Excitation in Phase Diffusive Regime

Saxon Liou and Watson Kuo
Department of Physics, National Chung Hsing University, Taichung
Taiwan

1. Introduction

Superconductivity exhibits elegant macroscopic quantum coherence in such a way that the many-body physics can be understood in a one-body way, described by the superconducting phase, and its quantum conjugate variable, the charge. When two superconductors are connected to each other through a tunnel junction, the charge tunneling can be controlled by the phase difference ϕ, leading to many interesting phenomena. For decades, because of the robustness of phase coherence in large junctions, a simple classical approach by modeling the system by a damped pendulum to the problem is successful and overwhelmed.(Tinkham 1996) However, as the sub-micro fabrication techniques had emerged in the 1990's, ultra small junctions were found exhibiting stronger quantum fluctuations in phase due to charging effect, which cannot be overlooked. For exploring the novel phenomena in the opposite limit, people have made devices with robust charging effect. These phenomena can be well understood by treating the charge tunneling as a non-coherent perturbation to the quantum states described by charge. However, in the case when the Josephson energy and charging energy are competing, neither approach gives a satisfactory description. One of the attempts is to include the coherent nature in the charge tunneling processes by introducing a phase correlation function in time, which quantifies the robustness of the phase coherence.(Ingold & Nazarov 1991) The correlation function, which has been studied in many other fields, has a universal relation to the dissipation of the system, called fluctuation-dissipation theorem. Taken in this sense, dissipation is an important controlling parameter in the phase coherence robustness. If the environment impedance of the junction is much smaller than the quantum resistance (for Cooper pairs) $R_Q=h/4e^2$, the phase fluctuation is strongly damped, leading to pronounced phase coherence.(Devoret, Esteve et al. 1990) Indeed, why the classical model is so successful? The pioneer work done by Cadeira and Leggett(Caldeira & Leggett 1983) has pointed out that the environment impedance plays an important role. Their work stimulated many studies in dissipation-driven phase transitions in various systems including Josephson junctions.(Leggett, Chakravarty et al. 1987; Schon & Zaikin 1990)
In this article, we will focus on the responses of Josephson junctions under microwave irradiation. Theoretically the phenomenon can be explained by the phase dynamics under an ac driving. In section 2, we will start from a classical picture by considering a Langevin

equation for phase to explore different regimes for phase dynamics.(Koval, Fistul et al. 2004) In certain condition the phase demonstrates the mode locking of the internal and external frequencies, namely the phenomenon of Shapiro steps. In section 3, we will focus on the phase diffusive regime and begin with the phase correlation so as to derive the resulting charge tunneling rate.(Liou, Kuo et al. 2008) In section 4, we briefly introduce the approach by using the Bloch waves as the quantum sate basis. In section 5, the photon-assist tunneling(Tucker & Feldman 1985) will be briefly addressed and in section 6, we review some experiment works. Possible microwave detection application based on the above phenomena will be discussed.

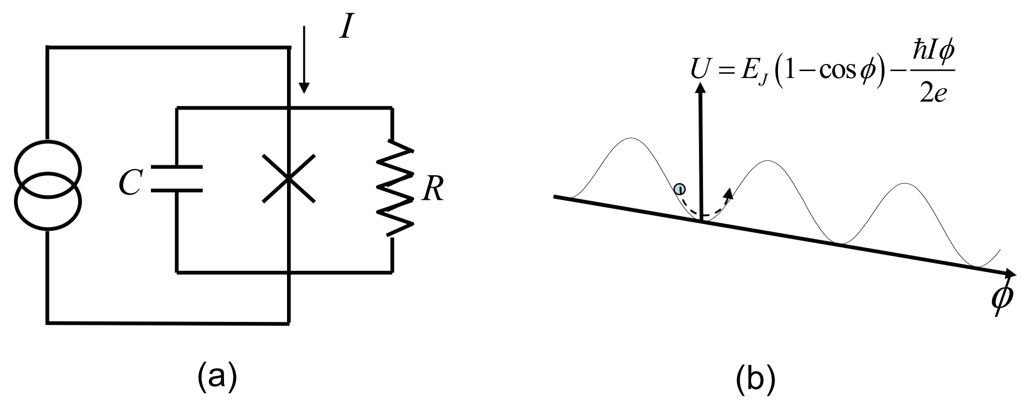


Fig. 1. (a) The model circuit of the resistively(R) and capacitively(C) shunted junction. (b) The phase dynamics in a junction under a dc current bias can be modeled with a particle of a mass C moving in a tilted washboard potential.

2. Langevin equation for superconducting phase

The celebrated resistively and capacitively shunted junction(RCSJ) is a classical way to describe the phase dynamics from a circuit point of view(see Fig. 1(a)). In this model, a model capacitance C and a model resistor R are connected in parallel to the junction under consideration. We note that a Josephson junction obeys the so-called voltage-phase relation and the current-phase relation that

$$V = \frac{\hbar}{2e}\dot{\phi}$$
$$I_S = I_C \sin\phi$$

in which I_C is the junction critical current. Therefore, the phase ϕ obeys an equation of motion as

$$\frac{\hbar}{8E_C}\ddot{\phi} + \frac{R_Q}{2\pi R}\dot{\phi} + \frac{I_C}{2e}\sin\phi = \frac{I(t)}{2e}, \qquad (1)$$

in which $E_C = \frac{e^2}{2C}$ is the charging energy, and I is the total bias current. The critical current is related to Josephson energy E_J through $I_C = 2e\frac{E_J}{\hbar}$.

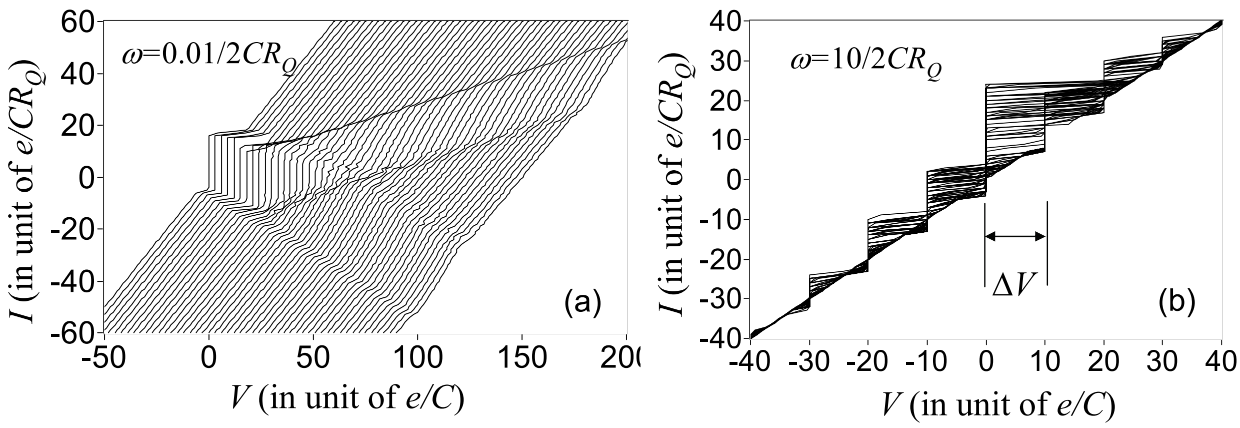


Fig. 2. The calculated current-voltage(IV) characteristics of an RC-shunted junction under the ac current driving of various amplitudes at a low frequency(a) and a high frequency(b). Note that the curves in (a) are shifted for clarity. With high frequency driving, the junction exhibits a mode-locking for superconducting phase at certain voltages with an equal spacing $\Delta V=\hbar\omega/2e$.

A dimensionless time scale, $\tau = \omega_p t$, in which $\omega_p = \frac{\sqrt{8E_C E_J}}{\hbar}$ is the plasma frequency, can be introduced further in such a way to yield a reduced equation, namely

$$\ddot{\phi} + \alpha\dot{\phi} + \sin\phi = f(\tau)\,. \tag{2}$$

Here $\alpha = \frac{2}{2\pi}\sqrt{\frac{2E_C}{E_J}}\frac{R_Q}{R}$ and $f = \frac{I}{I_C}$ is the bias current in unit of I_C. It can be clearly seen that expression (2) is of the form of Langevin equation describing a particle in a washboard potential as depicted in Fig. 1(b) with dimensionless dissipation α and a (dimensionless) driving force f. Also it resembles the dynamics of a damped simple pendulum, in which ϕ describes the angle of the pendulum to the vertical direction. When this system is kept in equilibrium with a heat bath, the heat bath provides thermal fluctuations which can be modeled as random forces to the system. In this sense, the right-hand side of the equation should be written as $f + \xi(\tau)$, in which $\xi(\tau)$ describes the random force and should obeys the fluctuation-dissipation theorem that $\langle \xi(\tau)\xi(0)\rangle = k_B T\alpha$ in which $\langle\cdots\rangle$ denotes ensemble average.

When the junction is excited by the microwaves, an external force periodic in time will be exerted to the particle, namely, $f(\tau) = f_0 + f_1 \sin\tilde{\omega}\tau$. Here $\tilde{\omega} = \omega/\omega_p$ is the dimensionless microwave frequency. By inputting all ingredients, one can solve equation (2) to get a distribution of particle in phase space $(\phi,\dot{\phi})$. The results can be compared to those from current-voltage (IV) measurement by calculating the average particle velocity $v = \langle\dot{\phi}\rangle = 2eV/\hbar\omega_p$ as a function of f_0 at various ac forces and dissipations. Intuitively speaking, when the junction when the junction is dc biased, it undergoes a periodic motion like the vertical circular motion

of a pendulum with an intrinsic angular frequency of $v=\langle\dot{\phi}\rangle$. Whenever there is an ac driving of which the frequency matches the intrinsic frequency, the junction exhibits interesting mode-locking phenomenon in such a way that the junction voltage is locked to the resonant condition. It turns out the mode-locking can occur at $V_n = n\hbar\omega/2e$, producing step structures in *IV* curves equally spaced in the voltage, called the Shapiro steps (see Fig. 2).
The above phenomenon is analyzed in a naïve way without dissipation consideration. Kovel et al (Koval, Fistul et al. 2004) explicitly derived the result without ac driving ($f_1 = 0$) analytically in the diffusive regime and found that at small voltage ranges, $I_S^0 = \frac{I_C}{\alpha}\frac{v}{v^2+\delta^2}$ in which δ quantifies the diffusion of the phase by $\langle\cos[\phi(\tau)-\phi(0)]\rangle = \exp(-\delta\tau)$. One can clear see that the dissipation gives rise to a finite voltage drop even in the "coherent" Cooper-pair tunneling branch. The supercurrent would be peaked at $v=\delta$, which can be verified by experiments.
With the presence of the ac driving, the current can be expressed by the summation of incoherent multi-photon absorption and emissions:

$$I_S = \sum_{n=-\infty}^{\infty} J_n^2\left(\frac{f_1}{\alpha\tilde{\omega}}\right) I_S^0(v-n\tilde{\omega}) = \sum_{n=-\infty}^{\infty} J_n^2\left(\frac{f_1}{\alpha\tilde{\omega}}\right) I_S^0\left(V-n\frac{\hbar\omega}{2e}\right) \tag{3}$$

Here $J_n(x)$ denotes the Bessel functions. We note that this result is similar to that derived by Tien and Gordon (Tien & Gordon 1963) for the single charge tunneling.

3. Spectral function theory

The incoherent Cooper-pair tunneling can be analyzed by the theory of spectral function $P(E)$, in which the tunneling rate can be expressed by

$$\Gamma(E) = \frac{\pi}{2\hbar} E_J{}^2 P(E), \tag{4}$$

from which the net supercurrent can be obtained $I_s(V) = 2e[\Gamma(2eV) - \Gamma(-2eV)]$ (Schon & Zaikin 1990). The spectral function is given by the Fourier transform of the correlation function(Devoret, Esteve et al. 1990; Ingold & Nazarov 1991)

$$P(E) = \frac{1}{2\pi\hbar}\int_{-\infty}^{\infty}\left\langle e^{i\varphi(t)}e^{-i\varphi(0)}\right\rangle e^{iEt/\hbar}dt\ .$$

In thermal equilibrium, the phase fluctuation is Gaussian so the Wick's theorem yields $\left\langle e^{i\phi(t)}e^{-i\phi(0)}\right\rangle = \exp K(t)$, in which $K(t) = \langle[\varphi(t)-\varphi(0)]\varphi(0)\rangle$. Applying the fluctuation-dissipation theorem, one can express $K(t)$ by using the environment impedance $Z(\omega)$ in unit of R_Q [10]:

$$K(t) = 2\int_{-\infty}^{\infty}\frac{d\omega}{\omega}\frac{\mathrm{Re}\,Z(\omega)}{R_Q}\frac{e^{-i\omega t}-1}{1-e^{-\hbar\omega/k_BT}}\ . \tag{5}$$

In the case of an Ohmic dissipation, $K(t)$ follows a square law in a short time scale. Whereas it linearly decreases with time $K(t) = -\left(R/R_Q\right)\left(\pi k_B T/\hbar\right)t$ beyond the RC time $\tau_{RC} \simeq \left(RC\right)^{-1}$ and inverse of Matsubara frequency $1/\nu_M = 2\pi k_B T/\hbar$, in which R and C are respectively the environmental impedance and capacitance. Here we can refer to the phase-diffusion result in previous section that $\delta \simeq \left(R/R_Q\right)\left(\pi k_B T/\hbar\omega_p\right)$. Also the RC time can be viewed as a mean-free time for the phase motion. One can see that this effect results in a finite resistance around zero-bias either in thermal fluctuation regime, $k_B T \gg \hbar\omega_p$ or in quantum fluctuation regime $R >\approx R_Q$.

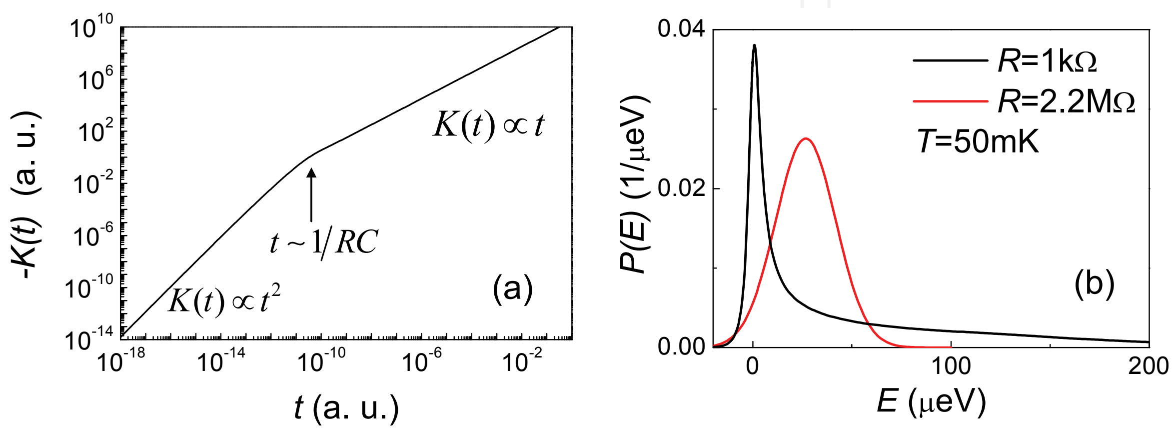


Fig. 3. (a) $K(t)$ function for an RC-shunted junction. When $t \ll 1/RC$, it follows a square law, describing a free particle moving under a constant driving force. In the long time limit, the particle moves in a diffusive way so as to yield a correlation function linear in time. (b)The calculated $P(E)$ function for $R<R_Q$ (black) and $R>R_Q$(red). For $R<R_Q$, the function is peaked at E=0 while for $R>R_Q$, the peak will be at $E=E_C$, signify a Coulomb-blockaded charge tunneling. Adopted from (Kuo, Wu et al. 2006)

In order to find the microwave influence to the supercurrent via the correlation function, we turn to the classical equation of motion of superconduting phase under an ac driving force by applying the model of a tilted-washboard potential. When the ac-bias frequency is much smaller than the plasma frequency, the system is in the linear response regime. It can be shown that the correlation function should have the following form, $K(t) = K_0(t) + K_{ac}(t)$, in which $K_0(t)$ is the correlation function in absence of microwave while $K_{ac}(t)$ represents the contribution due to ac-driving under different initial conditions. Since the superconducting phase obeys an equation of motion of $\dot{\varphi} = 2eV/\hbar$, for the ac-driving part, the phase would in general have a sinusoidal motion with an amplitude of $x = 2eV_{ac}/\hbar\omega$ and a frequency the same as the ac-driving one. Substitute the result into the correlation function, we have

$$\left\langle e^{i\varphi(t)} e^{-i\varphi(0)} \right\rangle \simeq \left\langle e^{i\varphi_0(t)} e^{-i\varphi_0(0)} \right\rangle \left\langle e^{ix\cos(\omega t+\theta) - ix\cos\theta} \right\rangle_\theta,$$

in which θ is the initial phase constant, and $\varphi_0(t)$ is the part of high frequency fluctuations. The latter average on the right-hand side is taken on the initial condition, θ. Now the $P(E)$ reads

$$P(E) = \lim_{T\to\infty} \frac{1}{2\pi\hbar T} \int_{-\infty}^{\infty}\int_{0}^{T} \left\langle e^{i\varphi_0(t)} e^{-i\varphi_0(0)} \right\rangle e^{ix\cos\omega(t+t')} e^{-ix\cos\omega t'} e^{iEt/\hbar} dt'dt$$
$$= \sum_{n=-\infty}^{\infty} J_n^2(x) P_0(E - n\hbar\omega) \tag{6}$$

Here $P_0(E)$ is the spectral function in absence of the microwave influence, and satisfies the detailed balance $P_0(-E) = \exp(-E/k_B T) P_0(E)$, a consequence of thermal equilibrium. We note that equation (6) is an expression of multiple photon absorption and emission with the amplitude of Bessel function $J_n(x)$.

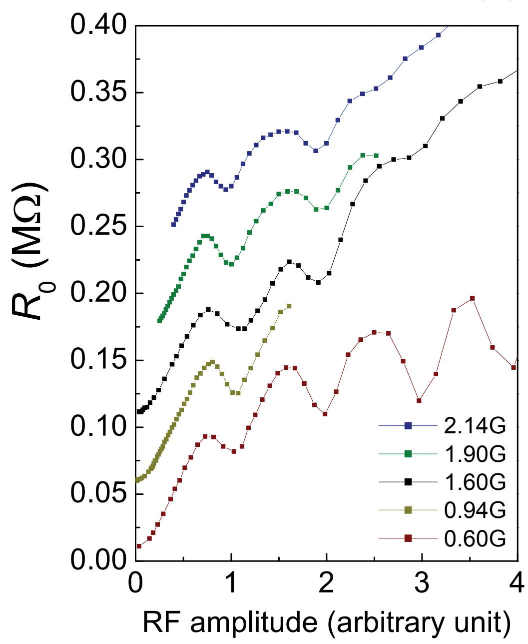


Fig. 4. The zero-bias resistance as a function of irradiating microwave (or RF) amplitude at various frequencies. Here the resistances are shifted and the microwave amplitude is rescaled to its period for each curve. The oscillation period is related to the superconducting gap of the electrodes. Adopted from (Liou, Kuo et al. 2008)

Expression (6) gives a supercurrent

$$I_s(V) = \sum_{n=-\infty}^{\infty} J_n^2(x) I_s^0\left(V - n\frac{\hbar\omega}{2e}\right). \tag{7}$$

Here $I_s^0(V)$ is the Cooper-pair tunneling current in absence of the microwaves. When the environmental impedance is much smaller than the quantum resistance, the spectral function becomes Delta-function so as to yield a coherent supercurrent, I_C at zero bias voltage. In turn, the microwave-induced supercurrent becomes $I_s(V_n) = I_C J_n^2(x)$ at a voltage $V_n = n\frac{\hbar\omega}{2e}$, leading to the structure of Shapiro steps. Ideally, each step in IV curves represents a constant-voltage state, labeled by n, featuring a "coherent" charge tunneling generated by the mode-locking. When the bias voltage is ramped, the junction would switch from one constant-voltage state to another, and eventually jumps to the finite-voltage state. It is noteworthy that in the analogy of a driven pendulum described in the previous section, the mode-locking should yield $I_s(V_n) = I_C |J_n(x)|$, a different result from the incoherent square dependence.

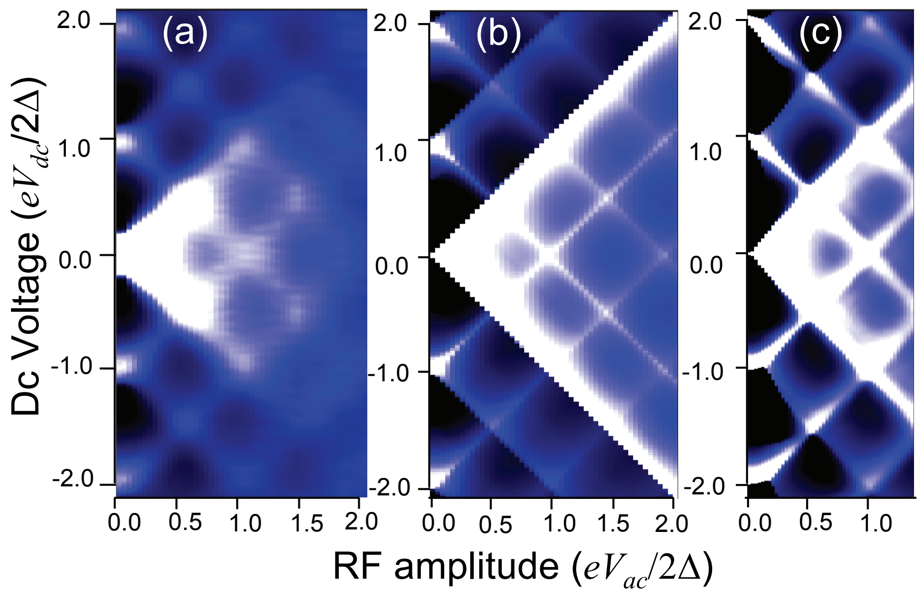


Fig. 5. The intensity plots of the dynamical conductance as a function of dc bias voltage V_{dc} and microwave amplitude V_{ac} of a long array(a) and a short array(c). According to the model described in text, the conductance peaks evolve into a "mesh" structure with the same period in V_{dc} and in V_{ac} of $2\Delta/e$. Adopted from(Liou, Kuo et al. 2008)

When the microwave frequency ω is small, argument x and n large, the summation over n can be replaced by an integration of $u = \cos^{-1}(n/x)$:

$$I_s = (2\pi)^{-1} \int_0^{2\pi} I_s^0 (V_{dc} + V_{ac} \cos u) \mathrm{d}u . \tag{8}$$

This expression is quite simple: It follows the same result as in the classical detector model. We note that Eq. (8) gives a general description for mesoscopic charge tunneling processes and should be applicable to both Cooper-pair tunneling and quasiparticle tunneling in the superconductive junction system.

4. Bloch wave formalism

Previous results are classical in nature. In a quantum point of view, the phase is not a function of time, but time-evolving quantum states. The un-biased single junction Hamiltonian can be expressed by

$$H_0 = 4E_C n^2 - E_J \cos\phi . \tag{9}$$

Here n is the charge number, obeying the commutation relation $[n, \phi] = i$. Because the potential is periodic in ϕ, the wavefunctions have the form of Bloch waves in lattices:

$$\Psi(\phi) = u_{k,s}(\phi)e^{ik\phi}$$

In which $u_{k,s}(\phi)$ is the envelope function for lattice momentum k and band index s. When there is a bias, an interaction term $H_I = (\hbar/2e)I\phi$ is added to the Hamiltonian, rendering the change of the lattice momentum and inter-band transitions.

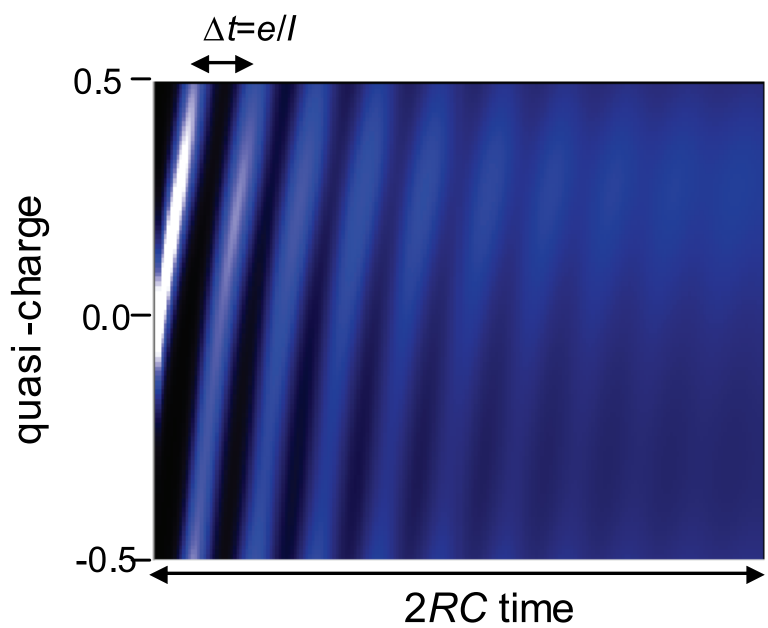


Fig. 6. The calculated diagonal elements of the single band density matrix for the junction under a dc-current bias. The expectation value of the lattice momentum (also called quasi-charge) linearly increases in time. This results in an oscillatory response with a period in time of e/I.

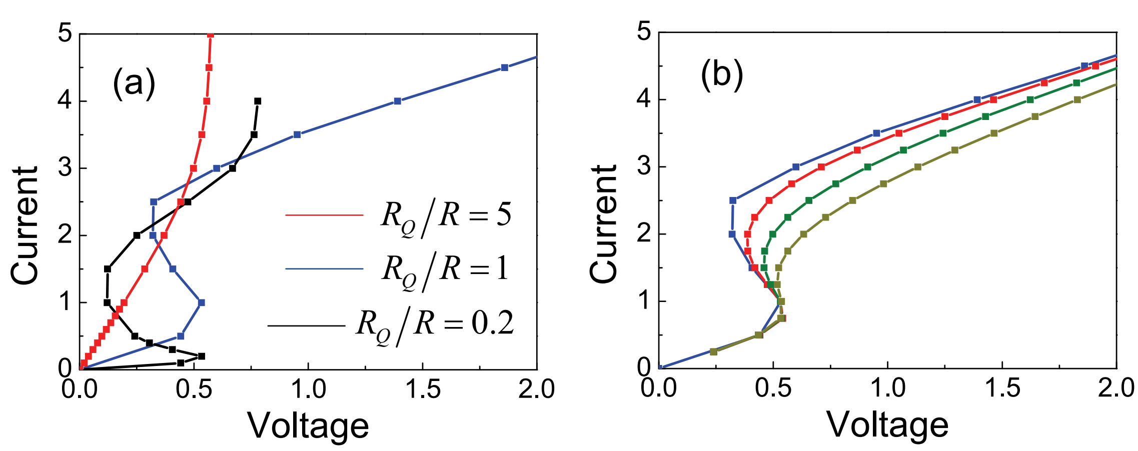


Fig. 7. (a) The calculated current-voltage characteristics of a junction with different dissipation strengths using the Bloch wave formalism. A Coulomb gap appears when the dissipation which quantified as R_Q/R is weak, featuring a relative stable quasi-charge. When the bias current is larger than $I_x = e/RC$, the quasi-charge starts to oscillate, turning the IV curve to a back-bending structure. (b) The IV curves of a R=R_Q junction under the ac driving of various amplitudes, I_1. In both figures the voltage is presented in unit of e/C while the current is presented in unit of e/R_QC.

It has been shown that in quantum dissipative system, the effect of environment can be introduced through a random bias and an effective damping to the system.(Weiss 2008) These effects would be better considered by using the concept of density matrix, ρ in stead of wavefunctions $\Psi(\phi)$:

$$\frac{\partial \rho}{\partial t} = \frac{i}{\hbar}[\rho, H].$$

Especially, when dropping the contributions from the off-diagonal elements, one can write down the differential equations for the diagonal parts:

$$\frac{\partial \sigma_s}{\partial t} = -\frac{I}{e}\frac{\partial \sigma_s}{\partial k} + \frac{1}{eR}\frac{\partial}{\partial k}\left(\sigma_s V_s\right) + \frac{k_B T}{e^2 R}\frac{\partial^2 \sigma_s}{\partial k^2} + \sum_{s'}\left\{-\left(\Gamma^Z_{ss'} + \Gamma^E_{ss'}\right)\sigma_s + \left(\Gamma^Z_{s's} + \Gamma^E_{s's}\right)\sigma_{s'}\right\} \tag{10}$$

Here $\sigma_s(k) = \rho_{kk,ss}$ denotes the diagonal element of the density matrix for quasi-momentum k and band s. Also called master equation, expression (10) describes the time evolution of the probability of state $|k,s\rangle$. The terms describe the effect of the external driving force, the resistive force, the random force, and interband transitions due to Zener tunneling($\Gamma^Z_{ss'}$-terms) and energy relaxation($\Gamma^E_{ss'}$-terms). Also, $V_s = \frac{1}{e}\frac{\partial E_{k,s}}{\partial k}$ describes the dispersion relation of the Bloch waves in band s.

By calculate the time-evolution of the density matrix elements, one can obtain the corresponding junction voltage $V = \sum_s \int V_s \sigma_s dk$ under a driving current I, yielding a comparison to the IV measurement results(Watanabe & Haviland 2001; Corlevi, Guichard et al. 2006). The most important feature of this approach is the Bloch oscillation under a constant bias current, $I = 2e\omega/2\pi$ as illustrated in Fig. 6. In the IV calculations, a Coulomb gap appears when $R_Q/R < 1$, featuring a relative stable quasi-charge. When the bias current is larger than $I_x = e/RC$, the driving force is large enough for the quasi-charge to oscillate. The Bloch oscillation features a back-bending structure in the IV curve which cannot be explained by previous approaches(see Fig. 7 for calculation results and Fig. 8 for experimental results).

When the junction is driven by the ac excitation, namely $I(t) = I_0 + I_1 \cos \omega t$, a mode-locking phenomenon may be raised at specific dc current $I_n = 2ne\omega/2\pi$. This mode-locking can be viewed as a counterpart of the Shapiro steps, which gives characteristic voltages $V_n = n\hbar\omega/eV$. The master equation approach, although more accurate than the classical ways, involves complicate calculations so a numerical method is un-evitable.

5. Photon-assisted tunnelling

The method introduced in previous section is a perturbative approach which may not be appropriate when the bias current is large. Alternatively, one may consider the eigen-energy problem for a periodically driven system describe by the Hamiltonian:

$$H = H_0 + H_I(t),$$

in which $H_0 = 4E_C n^2 - E_J \cos\phi$, and H_I is a periodic function with frequency ω, satisfying $H_I\left(t + \frac{2\pi}{\omega}\right) = H_I(t) = \sum_n H_{I,n} e^{in\omega t}$. Here $H_{I,n}$ is the n-th Fourier component in the frequency domain.
In general, it can be solved by applying the Floquet's theorem, which is similar to the Bloch theorem, in the following way:

$$\Psi(t) = \sum_n \psi_n e^{in\omega t} e^{-\frac{i}{\hbar}Et}. \tag{11}$$

The result can be viewed as a main level at energy E with sideband levels spaced by $\hbar\omega$. To determine the coefficients Ψ_n, one needs to solve the eigen equation:

$$H_0\psi_n + \sum_m H_{I,m}\psi_{n+m} = E\psi_n.$$

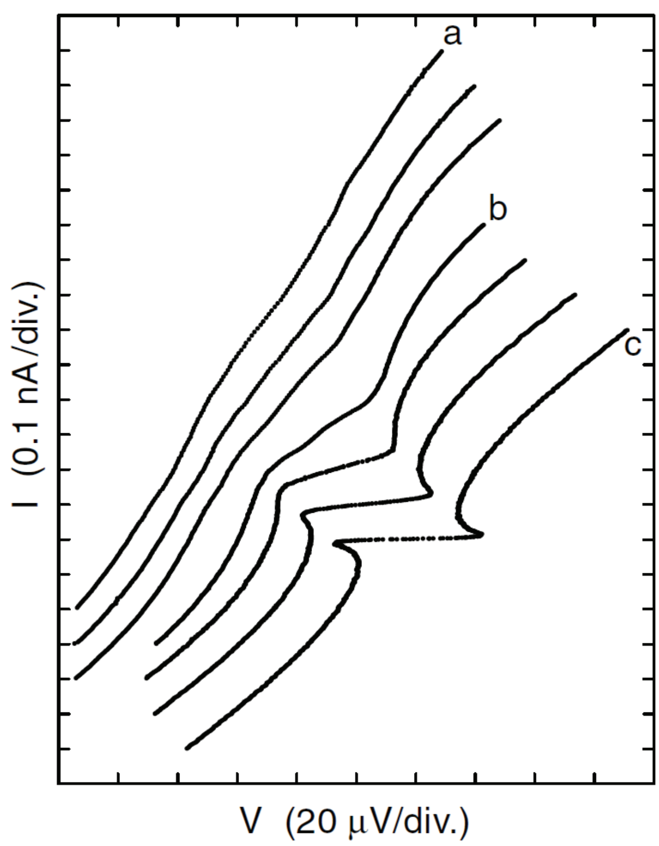


Fig. 8. The *IV* curves of the single junction in tunable environment of different impedances. From top left to bottom right, the environment impedance increases. Origin of each curve is offset for clarity. Adopted from (Watanabe & Haviland 2001)

Tien and Goldon (Tien & Gordon 1963) gave an simple model to describe the charge tunneling in the presence of microwaves. Suppose the ac driven force produces an ac modulation in the state energy that $E = E_0 + eV_{ac}\cos\omega t$ for an unperturbed wavefunction φ_0. Then the change in wavefunction is simply on the dynamical phase such that,

$$\begin{aligned}\Psi(t) &= \varphi_0 \exp\left[\frac{i}{\hbar}\int_0^t \left(E_0 + eV_{ac}\cos\omega t'\right)dt'\right] \\ &= \varphi_0 \sum_n J_n(x) e^{in\omega t} e^{-\frac{i}{\hbar}E_0 t}\end{aligned} \quad (12)$$

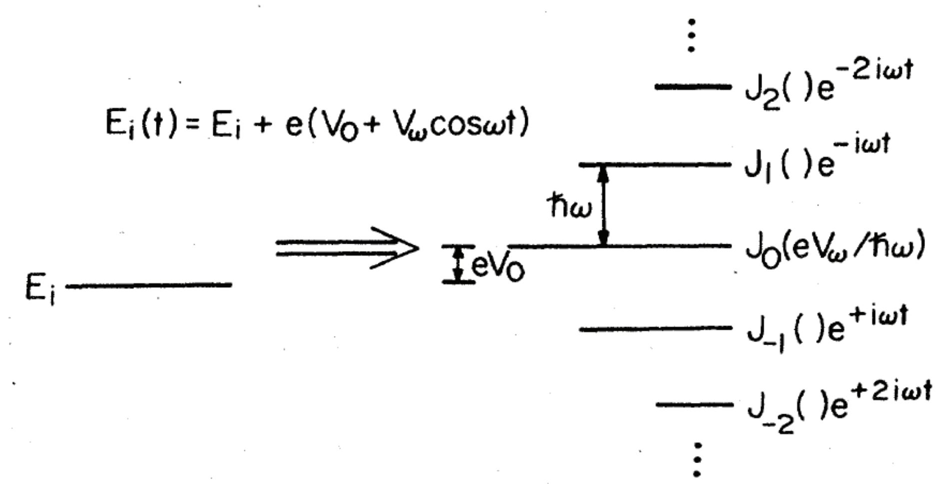


Fig. 9. The energy levels generated according to Eq. (12) in the presence of a microwave field. Adopted from (Tucker & Feldman 1985).

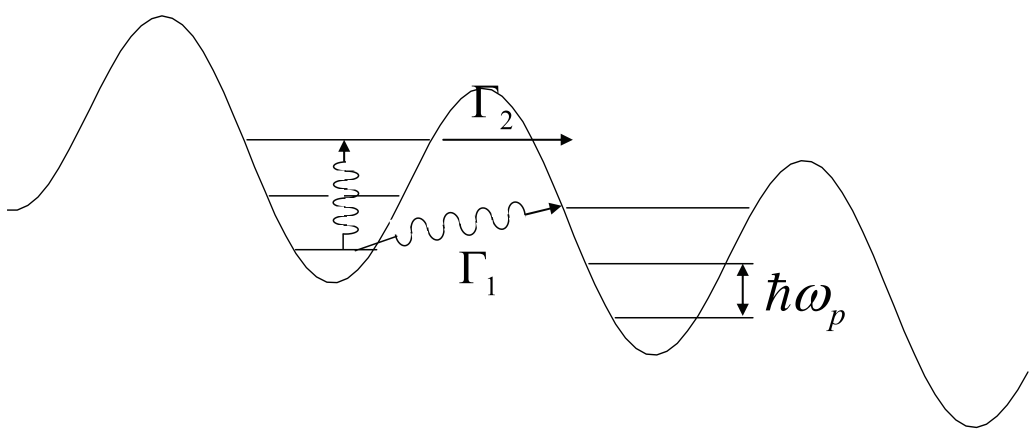


Fig. 10. A schematic of the enhanced macroscopic quantum tunneling in a single Josephson junction due to photon excitation. In each potential valley, the quantum states may form a harmonic oscillator ladder with a spacing of $\hbar\omega_p$. The absorption of photon energy may lead to an inter-valley resonant tunneling Γ_1, and a tunneling followed by an inner-valley excitation, Γ_2. The enhancement in macroscopic quantum tunneling results in a reduction of junction critical current.

Again $x = 2eV_{ac}/\hbar\omega$ as defined before. This expression is useful in the tunneling-Hamiltonian formalism applicable to the high-impedance devices such as single electron

transistors and quantum dots. One may include additional tunneling events through the side-band states with energies of $E_0 \pm n\hbar\omega$, namely the photon-assisted tunneling(PAT).
The PAT is a simple way to probe the quantum levels in the junctions. For example, if the Josephson coupling energy E_J is relatively large, a single potential valley of the washboard potential can be viewed as a parabolic one. In this case the system energy spectrum has a structure of simple harmonic ladder as shown in Fig. 10. The microwave excitation enhances the tunneling of the phase to adjacent valleys, also called macroscopic quantum tunneling when the photon energy matches the inter-level spacing. This can be observed in the reduction of junction critical current.

6. Experiments on ultra-small junctions

The Josephson junction under the microwave excitation has been studied for decades and much works have contributed to the topic, however mostly on low impedance junctions. (Tinkham 1996) Here the main focus is the junctions with small dissipation, namely, with environmental impedance $\mathrm{Re}\, Z \geq R_Q$. Although a large junction tunneling resistance as well as small junction capacitance can be obtained by using advanced sub-micron lithography, the realization of the high impedance condition remains a challenge to single junctions because of large parasitic capacitance between electrodes. Tasks have been made by using electrodes of high impedance to reduce the effective shunted resistance and capacitance.(Kuzmin & Haviland 1991) Another approach to this problem can be made by using systems in a moderate phase diffusive regime by thermal fluctuation, namely, $E_J \simeq k_B T$. Koval et al performed experiments on sub-micron Nb/AlO$_x$/Nb junctions and found a smooth and incoherent enhancement of Josephson phase diffusion by microwaves. (Koval, Fistul et al. 2004) This enhancement is manifested by a pronounced current peak at the voltage $V_p \propto \sqrt{P}$. Recently experiments on untrasmall Nb/Al/Nb long SNS junctions have found that the critical current increases when the ac frequency is larger than the inverse diffusion time in the normal metal, whereas the retrapping current is strongly modified when the excitation frequency is above the electron-phonon rate in the normal metal. (Chiodi, Aprili et al. 2009)
Double junctions, also called Bloch transistors and junction arrays are much easier for experimentalists to realize the high impedance (low dissipation) condition. The pioneer work by Eiles and Martinis provided the Shapiro step height versus the microwave amplitude in ultra-small double junctions.(Eiles & Martinis 1994) Several works found that the step height satisfies a square law, $I_s(V_n) = I_C J_n^2(x)$ instead of the RCSJ result, $I_s(V_n) = I_C \left|J_n(x)\right|$.(Eiles & Martinis 1994; Liou, Kuo et al. 2007) In the one-dimensional(1D) junction arrays, the supercurrent as a function of microwave amplitude can be found to obey $I_s(0) = I_C J_0^2(x)$ at high frequencies, $\hbar\omega > k_B T$, although no Shapiro steps were seen. At low frequencies, the current obeys the classical detector result as in expression (8) even for quasi-particle tunneling. (Liou, Kuo et al. 2008) Therefore a direct and primary detection scheme was proposed by using the 1D junction arrays.
In single junctions with a high environmental impedance, people has reported observation of structures in IV curves at $I = 2e\omega/2\pi$, featuring the Bloch oscillations due to pronounced charge blockade. (Kuzmin & Haviland 1991) The 1D arrays also demonstrate similar

interesting behavior signifying time-correlated single charge tunneling when driven by external microwaves. This behavior yields a junction current of $I_n = 2ne\omega/2\pi$ as what was found in the single junctions.(Delsing, Likharev et al. 1989; Andersson, Delsing et al. 2000) Recently, the Bloch oscillations are directly observed in the "quantronium" device and a current-to-frequency conversion was realized. (Nguyen, Boulant et al. 2007)

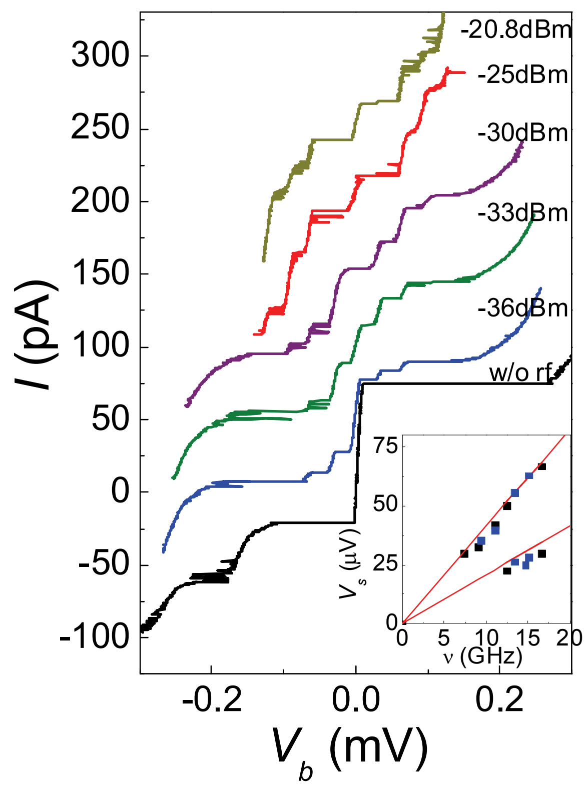


Fig. 11. The *IV* curves of a double junction under microwave irradiation clearly show Shapiro steps. Inset illustrate the step voltages obey the theoretical prediction. Adopted from (Liou, Kuo et al. 2007)

PAT is an ideal method to probe the quantum levels or band gaps in a quantum system. For the charge dominant system(E_C>E_J , R>R_Q) as an example, Flees et al. (Flees, Han et al. 1997) studied the reduction of critical current of a Bloch transistor under a microwave excitation. The lowest photon frequency corresponding to the band gap in the transistor was found to reduce as the gate voltage tuned to the energy degeneracy point for two charge states. In another work, Nakamura et al. biased the transistor at the Josephson-quasiparticle (JQP) point. The irradiating microwaves produced a photon-assisted JQP current at certain gate voltages, providing an estimation of the energy-level splitting between two macroscopic quantum states of charge coherently superposed by Josephson coupling.(Nakamura, Chen et al. 1997) For the phase dominant systems, enhanced macroscopic quantum tunneling were observed in system of single Josephson junction (Martinis, Devoret et al. 1985; Clarke, Cleland et al. 1988) and superconducting quantum interference devices(SQUIDs)(Friedman, Patel et al. 2000; van der Wal, Ter Haar et al. 2000). Recently, devices based on Josephson junctions, such as SQUIDs, charge boxes, and single junctions have been demonstrated as an ideal artificial two-level system for quantum computation applications by using the microwave spectrometry.(Makhlin, Schon et al. 2001)

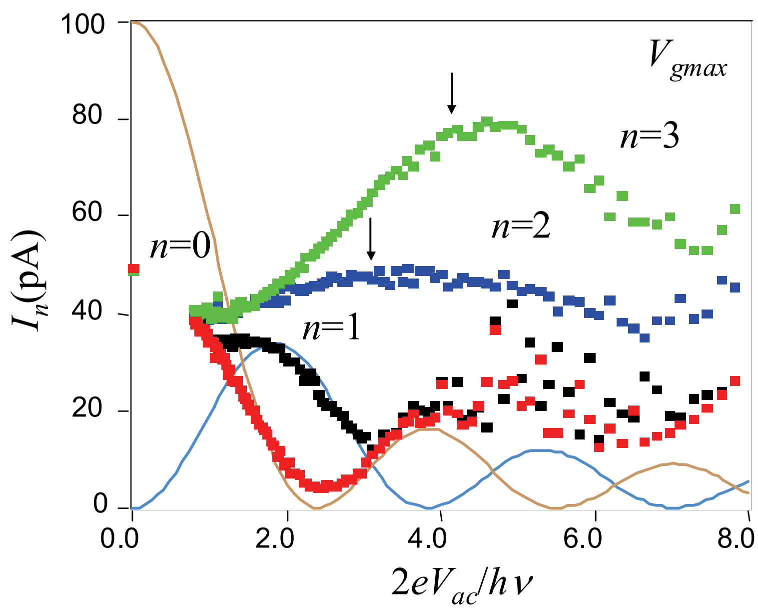


Fig. 12. The Shapiro height as a function of the microwave amplitude V_{ac} observed in a double junction system obeys the square law, a feature of incoherent photon absorption in this system. Adopted from(Liou, Kuo et al. 2007).

7. Conclusion

We have discussed the dc response of a Josephson junction under the microwave excitation in the phase diffusion regime theoretically as well as summarized recent experimental findings. In relative low impedance cases, the classical description (in phase) is plausible to explain the observed Shapiro steps and incoherent photon absorption. The quantum mechanical approaches may provide a more precise description for the experimental results of higher impedance cases such as Bloch oscillations and photon-assisted tunneling. In extremely high impedance cases, single charge tunneling prevails and a classical description in charge, such as charging effect can be an ideal approach.

8. Acknowledgement

The authors thank National Chung Hsing University and the Taiwan National Science Council Grant NSC-96-2112-M-005-003-MY3 for the support of this research.

9. References

Andersson, K., P. Delsing, et al. (2000). "Synchronous Cooper pair tunneling in a 1D-array of Josephson junctions." Physica B: Physics of Condensed Matter 284: 1816-1817.

Caldeira, A. and A. Leggett (1983). "Dynamics of the dissipative two-level system." Ann Phys 149: 374.

Chiodi, F., M. Aprili, et al. (2009). "Evidence for two time scales in long SNS junctions." Physical Review Letters 103(17): 177002.

Clarke, J., A. Cleland, et al. (1988). "Quantum mechanics of a macroscopic variable: the phase difference of a Josephson junction." Science 239(4843): 992.

Corlevi, S., W. Guichard, et al. (2006). "Phase-Charge Duality of a Josephson Junction in a Fluctuating Electromagnetic Environment." Physical Review Letters 97(9): 096802.

Delsing, P., K. K. Likharev, et al. (1989). "Time-correlated single-electron tunneling in one-dimensional arrays of ultrasmall tunnel junctions." Physical Review Letters 63(17): 1861.

Devoret, M., D. Esteve, et al. (1990). "Effect of the electromagnetic environment on the Coulomb blockade in ultrasmall tunnel junctions." Physical Review Letters 64(15): 1824-1827.

Eiles, T. M. and J. M. Martinis (1994). "Combined Josephson and charging behavior of the supercurrent in the superconducting single-electron transistor." Physical Review B 50(1): 627.

Flees, D. J., S. Han, et al. (1997). "Interband Transitions and Band Gap Measurements in Bloch Transistors." Physical Review Letters 78(25): 4817.

Friedman, J. R., V. Patel, et al. (2000). "Quantum superposition of distinct macroscopic states." Nature 406(6791): 43-46.

Ingold, G.-L. and Y. V. Nazarov (1991). *Single Charge Tunneling*. H. Grabert and M. H. Devoret. New York, Plenum. 294.

Koval, Y., M. Fistul, et al. (2004). "Enhancement of Josephson phase diffusion by microwaves." Physical Review Letters 93(8): 87004.

Kuo, W., C. S. Wu, et al. (2006). "Parity effect in a superconducting island in a tunable dissipative environment." Physical Review B 74(18): 184522-184525.

Kuzmin, L. S. and D. B. Haviland (1991). "Observation of the Bloch oscillations in an ultrasmall Josephson junction." Physical Review Letters 67(20): 2890.

Leggett, A., S. Chakravarty, et al. (1987). "Dynamics of the dissipative two-state system." Reviews of Modern Physics 59(1): 1-85.

Liou, S., W. Kuo, et al. (2007). "Shapiro Steps Observed in a Superconducting Single Electron Transistor." Chinese Journal of Physics 45: 230.

Liou, S., W. Kuo, et al. (2008). "Phase diffusions due to radio-frequency excitations in one-dimensional arrays of superconductor/ insulator/superconductor junctions." New Journal of Physics(7): 073025.

Makhlin, Y., G. Schon, et al. (2001). "Quantum-state engineering with Josephson-junction devices." Reviews of Modern Physics 73(2): 357.

Martinis, J. M., M. H. Devoret, et al. (1985). "Energy-Level Quantization in the Zero-Voltage State of a Current-Biased Josephson Junction." Physical Review Letters 55(15): 1543.

Nakamura, Y., C. Chen, et al. (1997). "Spectroscopy of energy-level splitting between two macroscopic quantum states of charge coherently superposed by Josephson coupling." Physical Review Letters 79(12): 2328-2331.

Nguyen, F., N. Boulant, et al. (2007). "Current to Frequency Conversion in a Josephson Circuit." Physical Review Letters 99(18): 187005.

Schon, G. and A. Zaikin (1990). "Quantum coherent effects, phase transitions, and the dissipative dynamics of ultra small tunnel junctions." Physics Reports 198: 237-412.

Tien, P. and J. Gordon (1963). "Multiphoton process observed in the interaction of microwave fields with the tunneling between superconductor films." Physical Review 129(2): 647-651.

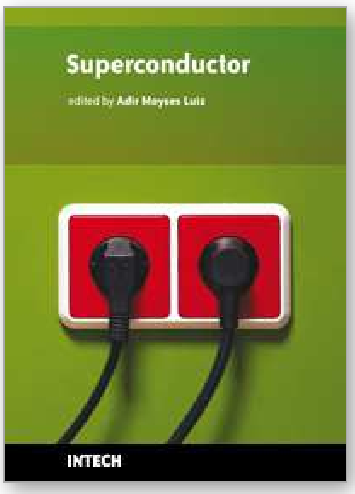
Tinkham, M. (1996). *Introduction to Superconductivity*. New York, McGraw-Hill.

Tucker, J. and M. Feldman (1985). "Quantum detection at millimeter wavelengths." Reviews of Modern Physics 57(4): 1055-1113.

van der Wal, C., A. Ter Haar, et al. (2000). "Quantum superposition of macroscopic persistent-current states." Science 290(5492): 773.

Watanabe, M. and D. Haviland (2001). "Coulomb Blockade and Coherent Single-Cooper-Pair Tunneling in Single Josephson Junctions." Physical Review Letters 86(22): 5120-5123.

Weiss, U. (2008). Quantum dissipative systems, World Scientific Pub Co Inc.



Superconductor
Edited by Doctor Adir Moyses Luiz

ISBN 978-953-307-107-7
Hard cover, 344 pages
Publisher Sciyo
Published online 18, August, 2010
Published in print edition August, 2010

This book contains a collection of works intended to study theoretical and experimental aspects of superconductivity. Here you will find interesting reports on low-Tc superconductors (materials with Tc< 30 K), as well as a great number of researches on high-Tc superconductors (materials with Tc> 30 K). Certainly this book will be useful to encourage further experimental and theoretical researches in superconducting materials.