Stochastic improvement of structural design

Soprano Alessandro and Caputo Francesco
Second University of Naples
Italy

1. Introduction

It is well understood nowadays that design is not an one-step process, but that it evolves along many phases which, starting from an initial idea, include drafting, preliminary evaluations, trial and error procedures, verifications and so on. All those steps can include considerations that come from different areas, when functional requirements have to be met which pertain to fields not directly related to the structural one, as it happens for noise, environmental prescriptions and so on; but even when that it's not the case, it is very frequent the need to match against opposing demands, for example when the required strength or stiffness is to be coupled with lightness, not to mention the frequently encountered problems related to the available production means.

All the previous cases, and the many others which can be taken into account, justify the introduction of particular design methods, obviously made easier by the ever-increasing use of numerical methods, and first of all of those techniques which are related to the field of mono- or multi-objective or even multidisciplinary optimization, but they are usually confined in the area of deterministic design, where all variables and parameters are considered as fixed in value. As we discuss below, the random, or stochastic, character of one or more parameters and variables can be taken into account, thus adding a deeper insight into the real nature of the problem in hand and consequently providing a more sound and improved design.

Many reasons can induce designers to study a structural project by probabilistic methods, for example because of uncertainties about loads, constraints and environmental conditions, damage propagation and so on; the basic methods used to perform such analyses are well assessed, at least for what refers to the most common cases, where structures can be assumed to be characterized by a linear behaviour and when their complexity is not very great.

Another field where probabilistic analysis is increasingly being used is that related to the requirement to obtain a product which is 'robust' against the possible variations of manufacturing parameters, with this meaning both production tolerances and the settings of machines and equipments; in that case one is looking for the 'best' setting, i.e. that which minimizes the variance of the product against those of design or control variables.

A very usual case – but also a very difficult to be dealt – is that where it is required to take into account also the time variable, which happens when dealing with a structure which degrades because of corrosion, thermal stresses, fatigue, or others; for example, when studying very light structures, such as those of aircrafts, the designer aims to ensure an assigned life to them, which are subjected to random fatigue loads; in advanced age the

aircraft is interested by a WFD (Widespread Fatigue Damage) state, with the presence of many cracks which can grow, ultimately causing failure. This case, which is usually studied by analyzing the behaviour of significant details, is a very complex one, as one has to take into account a large number of cracks or defects, whose sizes and locations can't be predicted, aiming to delay their growth and to limit the probability of failure in the operational life of the aircraft within very small limits (about $10^{-7}\pm10^{-9}$).
The most widespread technique is a 'decoupled' one, in the sense that a forecast is introduced by one of the available methods about the amount of damage which will probably take place at a prescribed instant and then an analysis in carried out about the residual strength of the structure; that is because the more general study which makes use of the stochastic analysis of the structure is a very complex one and still far away for the actual solution methods; the most used techniques, as the first passage theory, which claim to be the solution, are just a way to move around the real problems.
In any case, the probabilistic analysis of the structure is usually a final step of the design process and it always starts on the basis of a deterministic study which is considered as completed when the other starts. That is also the state that will be considered in the present chapter, where we shall recall the techniques usually adopted and we shall illustrate them by recalling some case studies, based on our experience.
For example, the first case which will be illustrated is that of a riveted sheet structure of the kind most common in the aeronautical field and we shall show how its study can be carried out on the basis of the considerations we introduced above.
The other cases which will be presented in this paper refer to the probabilistic analysis and optimization of structural details of aeronautical as well as of automotive interest; thus, we shall discuss the study of an aeronautical panel, whose residual strength in presence of propagating cracks has to be increased, and with the study of an absorber, of the type used in cars to reduce the accelerations which act on the passengers during an impact or road accident, and whose design has to be improved. In both cases the final behaviour is influenced by design, manufacturing process and operational conditions.

2. General methods for the probabilistic analysis of structures

If we consider the n-dimensional space defined by the random variables which govern a generic problem ("design variables") and which consist of geometrical, material, load, environmental and human factors, we can observe that those sets of coordinates (**x**) that correspond to failure define a domain (the 'failure domain' Ω_f) in opposition to the remainder of the same space, that is known as the 'safety domain' (Ω_s) as it corresponds to survival conditions.
In general terms, the probability of failure can be expressed by the following integral:

$$P_f = \int_{\Omega_f} f(\mathbf{x})\cdot d\mathbf{x} = \int_{\Omega_f} f_{x_1 x_2 \cdots x_n}(x_1, x_2, \cdots x_n)\cdot dx_1 dx_2 \cdots dx_n \tag{1}$$

where f_i represents the joint density function of all variables, which, in turn, may happen to be also functions of time. Unfortunately that integral cannot be solved in a closed form in most cases and therefore one has to use approximate methods, which can be included in one of the following typologies:
1) methods that use the limit state surface (LSS, the surface that constitutes the boundary of the failure region) concept: they belong to a group of techniques that model variously the

LSS in both shape and order and use it to obtain an approximate probability of failure; among these, for instance, particularly used are FORM (First Order Reliability Method) and SORM (Second Order Reliability Method), that represent the LSS respectively through the hyper-plane tangent to the same LSS at the point of the largest probability of occurrence or through an hyper-paraboloid of rotation with the vertex at the same point.
2) Simulation methodologies, which are of particular importance when dealing with complex problems: basically, they use Monte-Carlo (MC) technique for the numerical evaluation of the integral above and therefore they define the probability of failure on a frequency basis.
As pointed above, it is necessary to use a simulation technique to study complex structures, but in the same cases each trial has to be carried out through a numerical analysis (for example by FEM); if we couple that circumstance with the need to perform a very large number of trials, which is the case when dealing with very small probabilities of failure, very large runtimes are obtained, which are really impossible to bear. Therefore different means have been introduced in recent years to reduce the number of trials and to make acceptable the simulation procedures.
In this section, therefore, we resume briefly the different methods which are available to carry out analytic or simulation procedures, pointing out the difficulties and/or advantages which characterize them and the particular problems which can arise in their use.

2.1 LSS-based analytical methods

Those methods come from an idea by Cornell (1969), as modified by Hasofer and Lind (1974) who, taking into account only those cases where the design variables could be considered to be normally distributed and uncorrelated, each defined by their mean value μ_I and standard deviation σ_I, modeled the LSS in the standard space, where each variable is represented through the corresponding standard variable, i.e.

$$u_i = \frac{x_i - \mu_i}{\sigma_i} \tag{2}$$

If the LSS can be represented by a hyperplane (fig. 1), it can be shown that the probability of failure is related to the distance β of LSS from the origin in the standard space and therefore is given by

$$P_{fFORM} = 1 - \Phi(\beta) = \Phi(-\beta) \tag{3}$$

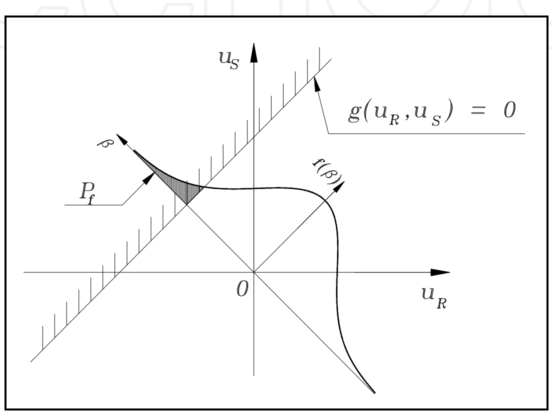


Fig. 1. Probability of failure for a hyperplane LSS

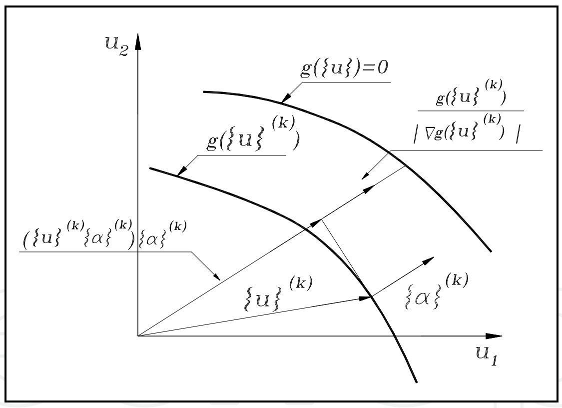


Fig. 2. The search for the design point according to RF's method

It can be also shown that the point of LSS which is located at the least distance β from the origin is the one for which the elementary probability of failure is the largest and for that reason it is called the maximum probability point (MPP) or the design point (DP).
Those concepts have been applied also to the study of problems where the LSS cannot be modeled as an hyperplane; in those cases the basic methods try to approximate the LSS by means of some polynomial, mostly of the first or the second degree; broadly speaking, in both cases the technique adopted uses a Taylor expansion of the real function around some suitably chosen point to obtain the polynomial representation of the LSS and it is quite obvious to use the design point to build the expansion, as thereafter the previous Hasofer and Lind's method can be used.
It is then clear that the solution of such problems requires two distinct steps, i.e. the research of the design point and the evaluation of the probability integral; for example, in the case of FORM (First Order Reliability Method) the most widely applied method, those two steps are coupled in a recursive form of the gradient method (fig. 2), according to a technique introduced by Rackwitz and Fiessler (RF's method). If we represent the LSS through the function g(**x**) = 0 and indicate with α_I the direction cosines of the inward-pointing normal to the LSS at a point x_0, given by

$$\alpha_i = -\frac{1}{|\nabla g|_0}\left(\frac{\partial g}{\partial u_i}\right)_0 \tag{4}$$

starting from a first trial value of **u**, the kth n-uple is given by

$$\{\mathbf{u}\}_k = \left[\{\mathbf{u}\}_{k-1}^T \cdot \{\alpha\}_k + \frac{g(\{\mathbf{u}\}_{k-1})}{\nabla g(\{\mathbf{u}\}_{k-1})}\right] \cdot \{\alpha\}_k \tag{5}$$

thus obtaining the required design point within an assigned approximation; its distance from the origin is just β and then the probability of failure can be obtained through eq. 3 above.
One of the most evident errors which follow from that technique is that the probability of failure is usually over-estimated and that error grows as curvatures of the real LSS increase; to overcome that inconvenience in presence of highly non-linear surfaces, the SORM

(Second Order Reliability Method) was introduced, but, even with Tved's and Der Kiureghian's developments, its use implies great difficulties. The most relevant result, due to Breitung, appears to be the formulation of the probability of failure in presence of a quadratic LSS via FORM result, expressed by the following expression:

$$P_{fSORM} = \Phi(-\beta) \cdot \prod_{i=1}^{n-1} (1 - \beta \cdot \kappa_i)^{-1/2} = P_{fFORM} \cdot \prod_{i=1}^{n-1} (1 - \beta \cdot \kappa_i)^{-1/2} \tag{6}$$

where κ_I is the i-th curvature of the LSS; if the connection with FORM is a very convenient one, the evaluation of curvatures usually requires difficult and long computations; it is true that different simplifying assumptions are often introduced to make solution easier, but a complete analysis usually requires a great effort. Moreover, it is often disregarded that the above formulation comes from an asymptotic development and that consequently its result is so more approximate as β values are larger.

As we recalled above, the main hypotheses of those procedures are that the random variables are uncorrelated and normally distributed, but that is not the case in many problems; therefore, some methods have been introduced to overcome those difficulties.

For example, the usually adopted technique deals with correlated variables via an orthogonal transformation such as to build a new set of variables which are uncorrelated, using the well known properties of matrices. For what refers to the second problem, the current procedure is to approximate the behaviour of the real variables by considering dummy gaussian variables which have the same values of the distribution and density functions; that assumption leads to an iterative procedure, which can be stopped when the required approximation has been obtained: that is the original version of the technique, which was devised by Ditlevsen and which is called Normal Tail Approximation; other versions exist, for example the one introduced by Chen and Lind, which is more complex and which, nevertheless, doesn't bring any deeper knowledge on the subject.

At last, it is not possible to disregard the advantages connected with the use of the Response Surface Method, which is quite useful when dealing with rather large problems, for which it is not possible to forecast *a priori* the shape of the LSS and, therefore, the degree of the approximation required. That method, which comes from previous applications in other fields, approximate the LSS by a polynomial, usually of second degree, whose coefficients are obtained by Least Square Approximation or by DOE techniques; the procedure, for example according to Bucher and Burgund, evolves along a series of convergent trials, where one has to establish a center point for the i-th approximation, to find the required coefficients, to determine the design point and then to evaluate the new approximating center point for a new trial.

Beside those here recalled, other methods are available today, such as the Advanced Mean Value or the Correction Factor Method, and so on, and it is often difficult to distinguish their own advantages, but in any case the techniques which we outlined here are the most general and known ones; broadly speaking, all those methods correspond to different degree of approximation, so that their use is not advisable when the number of variables is large or when the expected probabilities of failure is very small, as it is often the case, because of the overlapping of the errors, which can bring results which are very far from the real one.

2.2 Simulation-based reliability assessment

In all those cases where the analytical methods are not to be relied on, for example in presence of many, maybe even not gaussian, variables, one has to use simulation methods to assess the reliability of a structure: about all those methods come from variations or developments of an 'original' method, whose name is Monte-Carlo method and which corresponds to the frequential (or *a posteriori*) definition of probability.

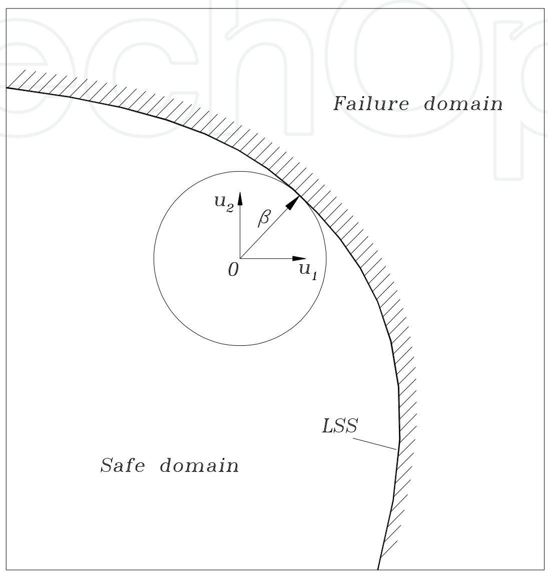


Fig. 3. Domain Restricted Sampling

For a problem with k random variables, of whatever distribution, the method requires the extraction of k random numbers, each of them being associated with the value of one of the variables via the corresponding distribution function; then, the problem is run with the found values and its result (failure of safety) recorded; if that procedure is carried out N times, the required probability, for example that corresponding to failure, is given by $P_f = n/N$, if the desired result has been obtained n times.

Unfortunately, broadly speaking, the procedure, which can be shown to lead to the 'exact' evaluation of the required probability if $N = \infty$, is very slow to reach convergence and therefore a large number of trials have to be performed; that is a real problem if one has to deal with complex cases where each solution is to be obtained by numerical methods, for example by FEM or others. That problem is so more evident as the largest part of the results are grouped around the mode of the result distribution, while one usually looks for probability which lie in the tails of the same distribution, i.e. one deals with very small probabilities, for example those corresponding to the failure of an aircraft or of an ocean platform and so on.

It can be shown, by using Bernouilli distribution, that if p is the 'exact' value of the required probability and if one wants to evaluate it with an assigned e_{max} error at a given confidence level defined by the bilateral protection factor k, the minimum number of trials to be carried out is given by

$$N_{min} = \left(\frac{2 \cdot k}{e_{max}} \right)^2 \frac{1-p}{p} \tag{7}$$

for example, if p = 10^{-5} and we want to evaluate it with a 10% error at the 95% confidence level, we have to carry out at least N_{min} = $1.537 \cdot 10^8$ trials, which is such a large number that usually larger errors are accepted, being often satisfied to get at least the order of magnitude of the probability.
It is quite obvious that various methods have been introduced to decrease the number of trials; for example, as we know that no failure point is to be found at a distance smaller than β from the origin of the axis in the standard space, Harbitz introduced the Domain Restricted Sampling (fig. 3), which requires the design point to be found first and then the trials are carried out only at distances from the origin larger than β; the Importance Sampling Method is also very useful, as each of the results obtained from the trials is weighted according to a function, which is given by the analyst and which is usually centered at the design point, with the aim to limit the number of trials corresponding to results which don't lie in the failure region.

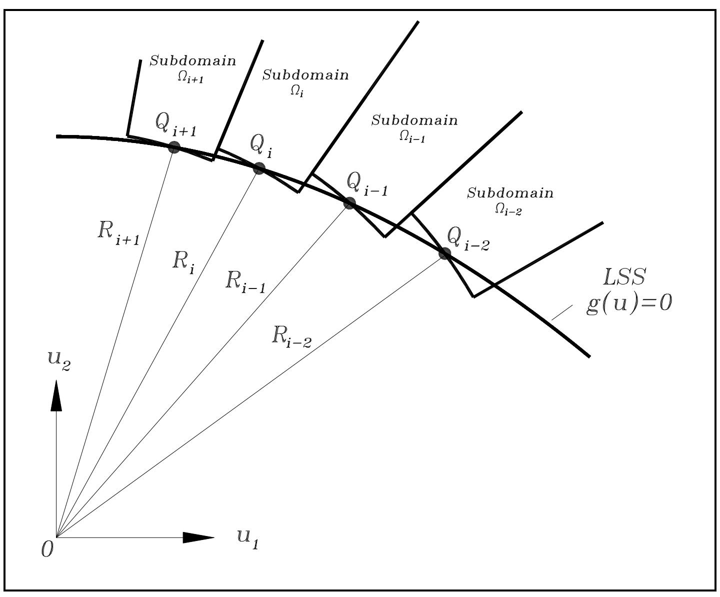


Fig. 4. The method of Directional Simulation

One of the most relevant technique which have been introduced in the recent past is the one known as Directional Simulation; in the version published by Nie and Ellingwood, the sample space is subdivided in an assigned number of sectors through radial hyperplanes (fig. 4); for each sector the mean distance of the LSF is found and the corresponding probability of failure is evaluated, the total probability being given by the simple sum of all results; in this case, not only the number of trials is severely decreased, but a better approximation of the frontier of the failure domain is achieved, with the consequence that the final probability is found with a good approximation.
Other recently appeared variations are related to the extraction of random numbers; those are, in fact, uniformly distributed in the 0-1 range and therefore give results which are rather clustered around the mode of the final distribution. That problem can be avoided if one resorts to use not really random distributions, as those coming from k-discrepancy theory, obtaining points which are better distributed in the sample space.
A new family of techniques have been introduced in the last years, all pertaining to the general family of *genetic algorithms*; that evocative name is usually coupled with an

imaginative interpretation which recalls the evolution of animal settlements, with all its content of selection, marriage, breeding and mutations, but it really covers in a systematic and reasoned way all the steps required to find the design point of an LSS in a given region of space. In fact, one has to define at first the size of the population, i.e. the number of sample points to be used when evaluating the required function; if that function is the distance of the design point from the origin, which is to be minimized, a selection is made such as to exclude from the following steps all points where the value assumed by the function is too large. After that, it is highly probable that the location of the minimum is between two points where the same function shows a small value: that coupling is what corresponds to marriage in the population and the resulting intermediate point represents the breed of the couple. Summing up the previous population, without the excluded points, with the breed, gives a new population which represents a new generation; in order to look around to observe if the minimum point is somehow displaced from the easy connection between parents, some mutation can be introduced, which corresponds to looking around the new-found positions.

It is quite clear that, besides all poetry related to the algorithm, it can be very useful but it is quite difficult to be used, as it is sensitive to all different choices one has to introduce in order to get a final solution: the size of the population, the mating criteria, the measure and the way of the introduction in breed of the parents' characters, the percentage and the amplitude of mutations, are all aspects which are to be the objects of single choices by the analyst and which can have severe consequences on the results, for example in terms of the number of generations required to attain convergence and of the accuracy of the method.

That's why it can be said that a general genetic code which can deal with all reliability problems is not to be expected, at least in the near future, as each problem requires specific cares that only the dedicated attentions of the programmer can guarantee.

3. Examples of analysis of structural details

An example is here introduced to show a particular case of stochastic analysis as applied to the study of structural details, taken from the authors' experience in research in the aeronautical field.

Because of their widespread use, the analysis of the behaviour of riveted sheets is quite common in aerospace applications; at the same time the interest which induced the authors to investigate the problems below is focused on the last stages of the operational life of aircraft, when a large number of fatigue-induced cracks appear at the same time in the sheets, before at least one of them propagates up to induce the failure of the riveted joint: the requirement to increase that life, even in presence of such a population of defects (when we say that a stage of Widespread Fatigue Damage, WFD, is taking place) compelled the authors to investigate such a scenario of a damaged structure.

3.1 Probabilistic behaviour of riveted joints

One of the main scopes of the present activity was devoted to the evaluation of the behaviour of a riveted joint in presence of damage, defined for example as a crack which, stemming from the edge of one of the holes of the joint, propagates toward the nearest one, therefore introducing a higher stress level, at least in the zone adjacent to crack tip.

It would be very appealing to use such easy procedures as compounding to evaluate SIF's for that case, which, as it is now well known, gives an estimate of the stress level which is built by reducing the problem at hand to the combination of simpler cases, for which the solution is known; that procedure is entirely reliable, but for those cases where singularities are so near to each other to develop an interaction effect which the method is not able to take into account.

Unfortunately, even if a huge literature is now available about edge cracks of many geometry, the effect of a loaded hole is not usually treated with the extent it deserves, may be for the particular complexity of the problem; for example, the two well known papers by Tweed and Rooke (1979; 1980) deal with the evaluation of SIF for a crack stemming from a loaded hole, but nothing is said about the effect of the presence of other loaded holes toward which the crack propagates.

Therefore, the problem of the increase of the stress level induced from a propagating crack between loaded holes could be approached only by means of numerical methods and the best idea was, of course, to use the results of FEM to investigate the case. Nevertheless, because of the presence of the external loads, which can alter or even mask the effects of loaded holes, we decided to carry out first an investigation about the behaviour of SIF in presence of two loaded holes.

The first step of the analysis was to choose which among the different parameters of the problem were to be treated as random variables.

Therefore a sort of sensitivity analysis was to be carried out; in our case, we considered a very specific detail, i.e. the space around the hole of a single rivet, to analyze the influence of the various parameters.

By using a Monte-Carlo procedure, some probability parameters were introduced according to experimental evidence for each of the variables in order to assess the required influence on the mean value and the coefficient of variation of the number of cycles before failure of the detail.

In any case, as pitch and diameter of the riveted holes are rather standardized in size, their influence was disregarded, while the sheet thickness was assumed as a deterministic parameter, varying between 1.2 and 4.8 mm; therefore, the investigated parameters were the stress level distribution, the size of the initial defect and the parameters of the propagation law, which was assumed to be of Paris' type.

For what refers to the load, it was supposed to be in presence of traction load cycles with R = 0 and with a mean value which followed a Gaussian probability density function around 60, 90 and 120 MPa, with a coefficient of variation varying according assigned steps; initial crack sizes were considered as normally distributed from 0.2 mm up to limits depending on the examined case, while for what concerns the two parameters of Paris' law, they were considered as characterized by a normal joint pdf between the exponent n and the logarithm of the other one.

Initially, an extensive exploration was carried out, considering each variable in turn as random, while keeping the others as constant and using the code NASGRO® to evaluate the number of cycles to failure; an external routine was written in order to insert the crack code in a M-C procedure. CC04 and TC03 models of NASGRO® library were adopted in order to take into account corner- as well as through-cracks. For all analyses 1,000 trials/point were carried out, as it was assumed as a convenient figure to be accepted to obtain rather stabilized results, while preventing the total runtimes from growing unacceptably long; the said M-C procedure was performed for an assigned statistics of one input variable at the time.

The results obtained can be illustrated by means of the following pictures and first of all of the fig. 5 where the dependence of the mean value of life from the mean amplitude of remote stress is recorded for different cases where the CV (coefficient of variation) of stress pdf was considered as being constant. The figure assesses the increase of the said mean life to failure in presence of higher CV of stress, as in this case rather low stresses are possible with a relatively high probability and they influence the rate of propagation in a higher measure than large ones.

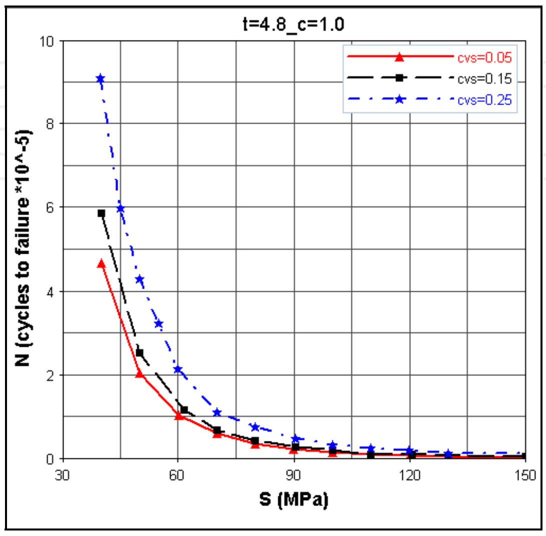


Fig. 5. Influence of the remote stress on the cycles to failure

In fig. 6 the influence of the initial geometry is examined for the case of a corner crack, considered to be elliptical in shape, with length c and depth a; a very interesting aspect of the consequences of a given shape is that for some cases the life for a through crack is longer than the one recorded for some deep corner ones; that case can be explained with the help of the plot of Fig. 7 where the growth of a through crack is compared with those of quarter corner cracks, recording times when a corner crack becomes a through one: as it is clarified in the boxes in the same picture, each point of the dashed curve references to a particular value of the initial depth.

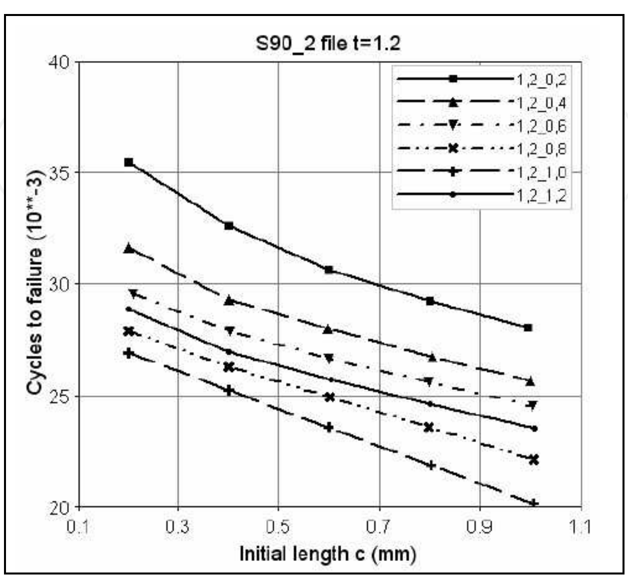


Fig. 6. Influence of the initial length of the crack on cycles to failure

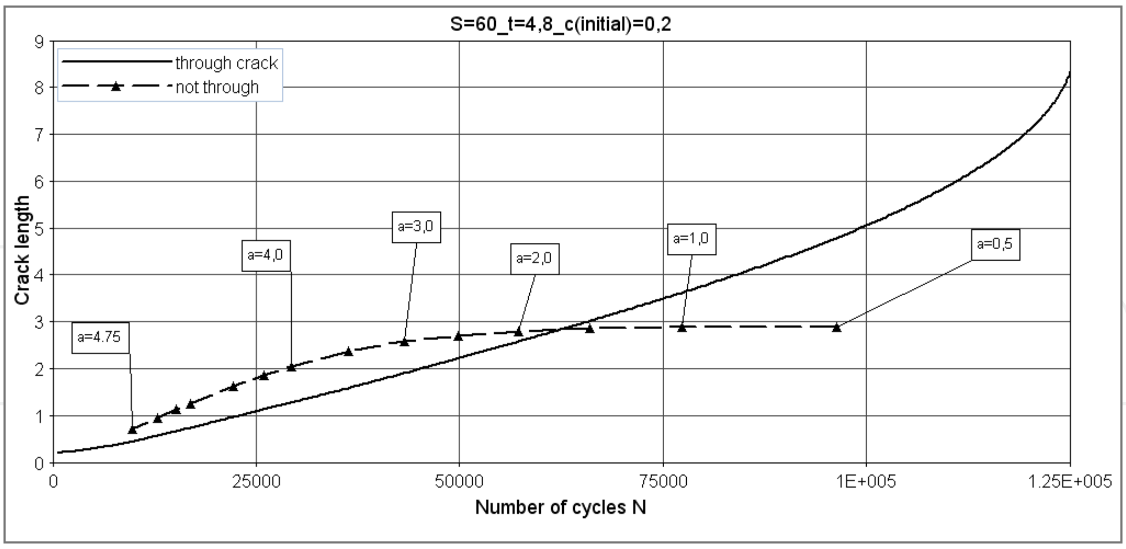


Fig. 7. Propagation behaviour of a corner and a through crack

It can be observed that beyond a certain value of the initial crack depth, depending on the sheet thickness, the length reached when the corner crack becomes a through one is larger than that obtained after the same number of cycles when starting with a through crack, and this effect is presumably connected to the bending effect of corner cracks.

For what concerns the influence exerted by the growth parameters, C and n according to the well known Paris' law, a first analysis was carried out in order to evaluate the influence of spatial randomness of propagation parameters; therefore the analysis was carried out considering that for each stage of propagation the current values of C and n were randomly extracted on the basis of a joint normal pdf between lnC and n. The results, illustrated in Fig. 8, show a strong resemblance with the well known experimental results by Wirkler.

Then an investigation was carried out about the influence of the same ruling parameters on the variance of cycles to failure. It could be shown that the mean value of the initial length has a little influence on the CV of cycles to failure, while on the contrary is largely affected by the CV of the said geometry. On the other hand, both statistical parameters of the distribution of remote stress have a deep influence on the CV of fatigue life.

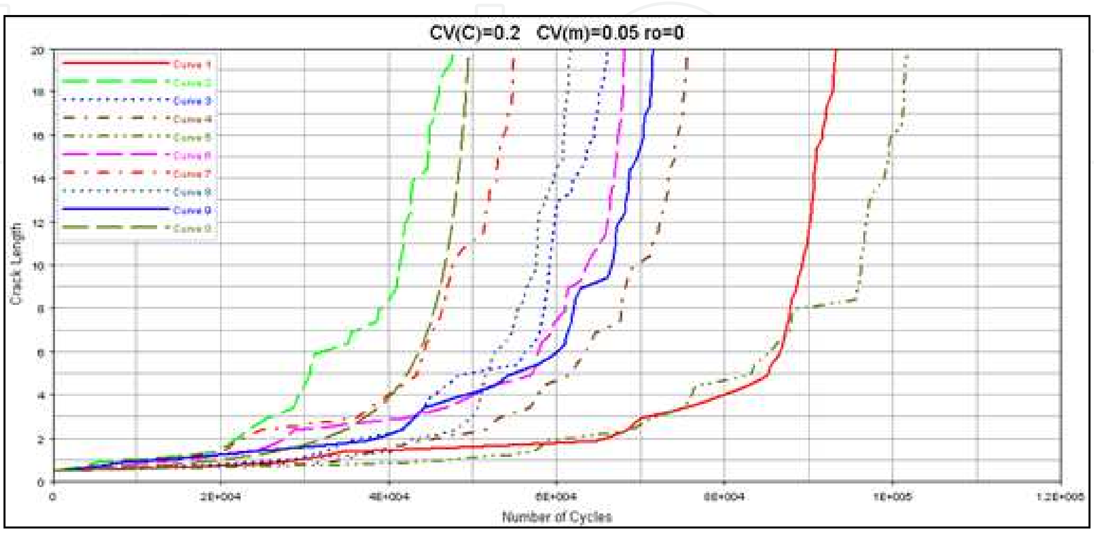


Fig. 8. Crack propagation histories with random parameters

Once the design variables were identified, the attention had to be focused on the type of structure that one wants to use as a reference; in the present case, a simple riveted lap joint for aeronautical application was chosen (fig. 9), composed by two 2024-T3 aluminium sheets, each 1 mm thick, with 3 rows of 10 columns of 5 mm rivets and a pitch of 25 mm. Several reasons suggest to analyze such a structure before beginning a really probabilistic study; for example, the state of stress induced into the component by external loads has to be evaluated and then it is important to know the interactions between existing singularities when a MSD (Multi-Site Damage) or even a WFD (Widespread Fatigue Damage) takes place. Several studies were carried out, in fact (for example, Horst, 2005), considering a probabilistic initiation of cracks followed by a deterministic propagation, on the basis that such a procedure can use very simple techniques, such as compounding (Rooke, 1986). Even if such a possibility is a very appealing one, as it is very fast, at least once the appropriate fundamental solutions have been found and recorded, some doubts arise when one comes to its feasibility.
The fundamental equation of compounding method is indeed as follows:

$$K = K^* + \sum\left(K_i - K^*\right) + K_e \tag{8}$$

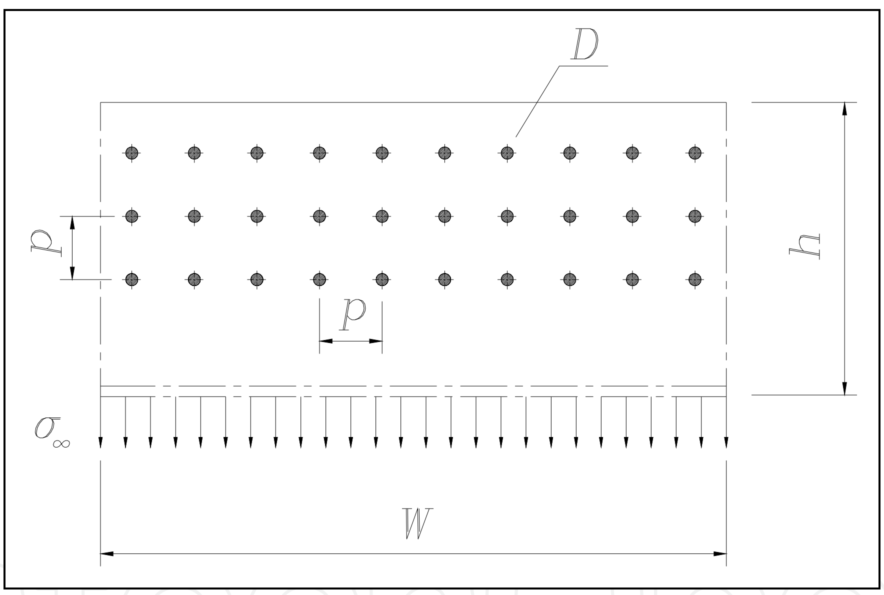


Fig. 9. The model used to study the aeronautical panel in WFD conditions

where the SIF at the crack tip of the crack we want to investigate is expressed by means of the SIF at the same location for the fundamental solution, K^*, plus the increase, with respect to the same 'fundamental' SIF, (K_i –K^*), induced by each other singularity, taken one at a time, plus the effect of interactions between existing singularities, still expressed as a SIF, K_e. As the largest part of literature is related to the case of a few cracks, the K_e term is usually neglected, but that assumption appears to be too weak when dealing with WFD studies, where the singularities approach each other; therefore one of the main reasons to carry out such deterministic analysis is to verify the extent of this approximation. It must be stressed that no widely known result is available for the case of rivet-loaded holes, at least for cases matching with the object of the present analysis; even the most known papers, which we quoted above deal with the evaluation of SIF for cracks which initiate on the edge of a

loaded hole, but it is important to know the consequence of rivet load on cracks which arise elsewhere.
Another aspect, related to the previous one, is the analysis of the load carried by each pitch as damage propagates; as the compliance of partially cracked pitches increases with damage, one is inclined to guess that the mean load carried by those zones decreases, but the nonlinearity of stresses induced by geometrical singularities makes the quantitative measure of such a variation difficult to evaluate; what's more, the usual expression adopted for SIF comes from fundamental cases where just one singularity is present and it is given as a linear function of remote stress. One has to guess if such a reference variable as the stress at infinity is still meaningful in WFD cases.
Furthermore, starting to study the reference structure, an appealing idea to get a fast solution can be to decompose the structure in simple and similar details, each including one pitch, to be analyzed separately and then added together, considering each of them as a finite element or better as a finite strip; that idea induces to consider the problem of the interactions between adjacent details.
In fact, even if the structure is considered to be a two-dimensional one, the propagation of damage in different places brings the consequence of varying interactions, for both normal and shearing stresses. For all reasons above, an extensive analysis of the reference structure is to be carried out in presence of different MSD scenarios; in order to get fast solutions, use can be made of the well known BEASY® commercial code, but different cases are to be verified by means of more complex models.
On the basis of the said controls, a wide set of scenarios could be explored, with two, three and also four cracks existing at a time, using a two-dimensional DBEM model; in the present case, a 100 MPa remote stress was considered, which was transferred to the sheet through the rivets according to a 37%, 26% and 37% distribution of load, as it is usually accepted in literature; that load was applied through an opportune pressure distribution on the edge of each hole. This model, however, cannot take into account two effects, i.e. the limited compliance of holes, due to the presence of rivets and the variations of the load carried by rivets mounted in cracked holes; both those aspects, however, were considered as not very relevant, following the control runs carried out by FEM.

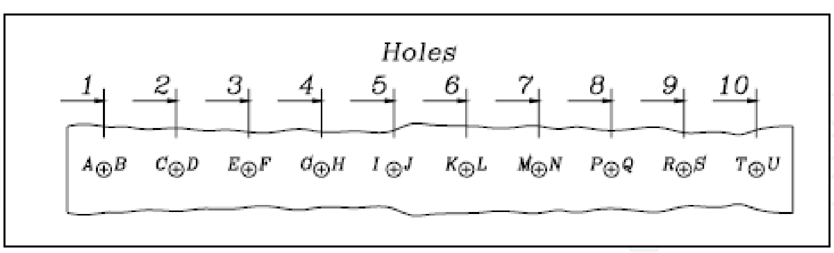


Fig. 10. The code used to represent WFD scenarios

For a better understanding of the following illustrations, one has to refer to fig. 10, where we show the code adopted to identify the cracks; each hole is numbered and each hole side is indicated by a capital letter, followed, if it is the case, by the crack length in mm; therefore, for example, E5J7P3 identifies the case when three cracks are present, the first, 5 mm long, being at the left side of the third hole (third pitch, considering sheet edges), another, 7 mm long, at the right side of the fifth hole (sixth pitch), and the last, 3 mm long, at the left side of the eight hole (eighth pitch).

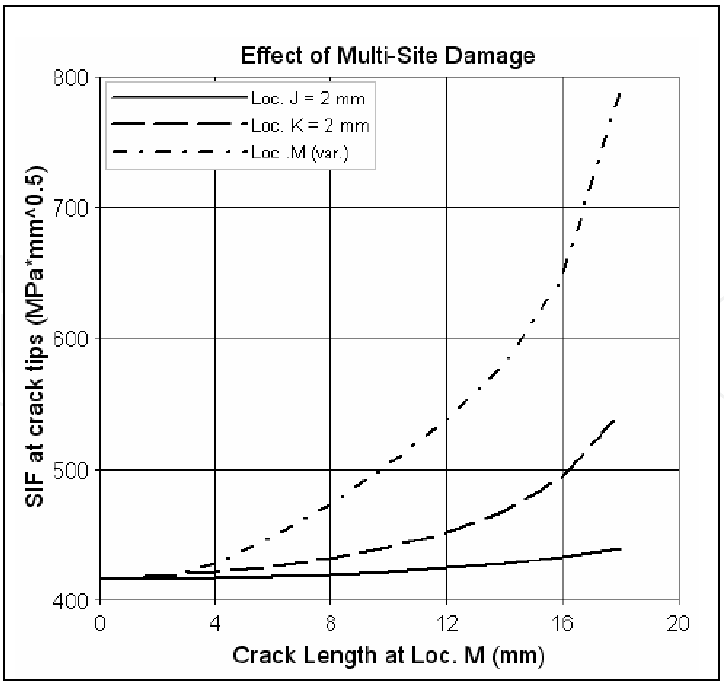


Fig. 11. Behaviour of J2K2Mx scenario

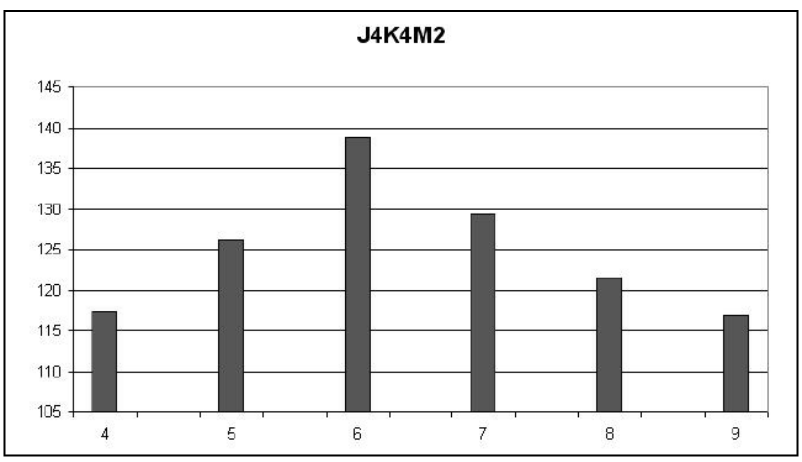


Fig. 12. Mean longitudinal stress loading different pitches for a 2 mm crack in pitch 7

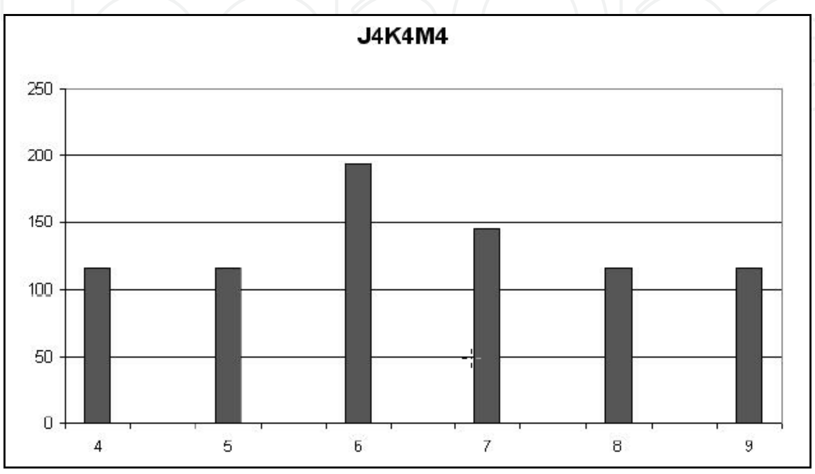


Fig. 13. Mean longitudinal stress loading different pitches for a 4 mm crack in pitch 7

In fig. 11 a three cracks scenario is represented, where in pitch 6 there are two cracks, each 2 mm long and another crack is growing at the right edge of the seventh hole, i.e. in the adjacent seventh pitch; if we consider only LEFM, we can observe that the leftmost crack (at location J) is not much influenced by the presence of the propagating crack at location M, while the central one exhibits an increase in SIF which can reach about 20%.

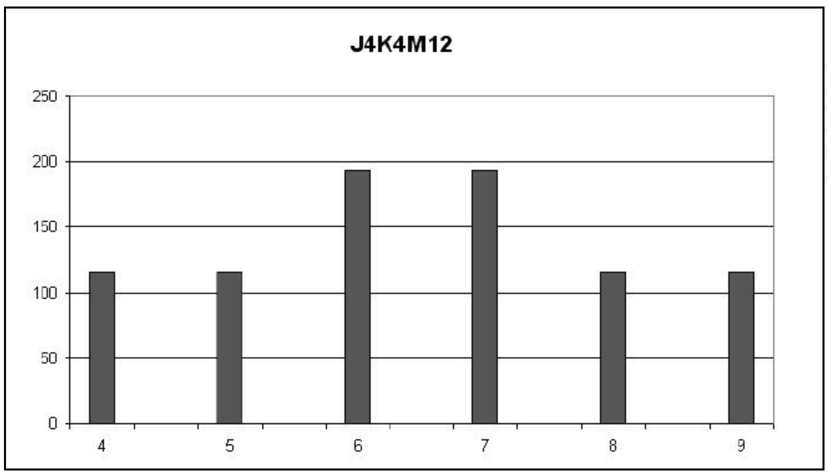


Fig. 14. Mean longitudinal stress loading different pitches for a 12 mm crack in pitch 7

The whole process can be observed by considering the mean longitudinal stress for different scenarios, as illustrated in Fig. 12, 13 and 14; in the first one, we can observe a progressive increase in the mean longitudinal stress around pitch no. 6, which is the most severely reduced and the influence of the small crack at location M is not very high.
As the length of crack in pitch 7 increases, however, the mean longitudinal stresses in both pitches 6 and 7 becomes quite similar and much higher of what is recorded in safe zones, where the same longitudinal stresses are not much increased in respect to what is recorded for a safe structure, because the transfer of load is distributed among many pitches.
The main results obtained through the previously discussed analysis can be summarized by observing that in complex scenarios high interactions exist between singularities and damaged zones, which can prevent the use of simple techniques such as compounding, but that the specific zone to be examined gets up to a single pitch beyond the cracked ones, of course on both sides. At the same time, as expected, we can observe that for WFD conditions, in presence of large cracks, the stress levels become so high that the use of LEFM can be made only from a qualitative standpoint.
If some knowledge about what to expect and how the coupled sheets will behave during the accumulation of damage has been obtained at this point of the analysis, we also realize, as pointed above, that no simple method can be used to evaluate the statistics of failure times, as different aspects will oppose and first of all the amount of the interactions between cracked holes; for that reason the only way which appears to be of some value is the direct M-C interaction as applied to the whole component, i.e. the evaluation of the 'true' history foe the sheets, to be performed the opportune number of times to extract reliable statistics; as the first problem the analyst has to overcome in such cases is the one related to the time consumption, it is of uttermost importance to use the most direct and quick techniques to obtain the desired results; for example, the use of DBEM coupled with an in-house developed code can give, if opportunely built, such guarantees.

In the version we are referring to, the structure was considered to be entirely safe at the beginning of each trial; then a damage process followed, which was considered as to be of Markow type. For the sake of brevity we shall not recall here the characters of such a process, which we consider to be widely known today; we simply mention that we have to define the initial scenario, the damage initiation criterion and the transitional probabilities for damage steps. In any case, we have to point out that other hypothesis could be assumed and first that of an initial damage state as related to EIFS (Equivalent Initial Flaw Size) or to the case of a rogue flaw, for example, don't imply any particular difficulty.
Two possible crack locations were considered at each hole, corresponding to the direction normal to the remote stress; the probability distribution of crack appearance in time was considered as lognormal, given by the following function:

$$f(N_i) = \frac{1}{\sigma_{ln} N_i \sqrt{2\pi}} \exp\left[-\frac{1}{2} \left(\frac{\ln(N_i) - \mu_{ln}}{\sigma_{ln}} \right)^2 \right] \tag{10}$$

with an immediate meaning of the different parameters; it has to be noted that in our case the experimental results available in literature were adapted to obtain P-S-N curves, in order to make the statistics dependent on the stress level. At each time of the analysis the extraction of a random number for each of the still safe locations was carried out to represent the probability of damage cumulated locally and compared with the probability coming from eq. (10) above; in the positive case, a new crack was considered as initiated in the opportune location.
In order to save time, the code started to perform the search only at a time where the probability to find at least one cracked location was not less than a quantity p chosen by the user; it is well known that, if p_f is the probability of a given outcome, the probability that the same outcome is found at least for one among n cases happening simultaneously is given by:

$$p = 1 - (1 - p_f)^n \; ; \tag{11}$$

in our case n is the number of possible locations, thus obtaining the initial analysis time, by inverting the probability function corresponding to eq. (11) above; in our trials it was generally adopted p = 0.005, which revealed to be a conservative choice, but of course other values could also be accepted. A particular choice had also to be made about the kind and the geometry of the initial crack; it is evident that to follow the damage process accurately a defect as small as possible has to be considered, for example a fraction of mm, but in that case some difficulties arise.
For example, such a small crack would fall in the range of *short cracks* and would, therefore, require a different treatment in propagation; in order to limit our analysis to a two-dimensional case we had to consider a crack which was born as a through one and therefore we choose it to be characterized by a length equal to the thickness of the sheet, i.e., 1.0 mm in our case.
Our choice was also justified by the fact that generally the experimental tests used to define the statistics represented in eq. (10) above record the appearance of a crack when the defect reaches a given length or, if carried out on drilled specimens, even match the initiation and the failure times, considering that in such cases the propagation times are very short. Given

an opportune integration step, the same random extraction was performed in correspondence of still safe locations, up to the time (cycle) when all holes were cracked; those already initiated were considered as propagating defects, integrating Paris-Erdogan's law on the basis of SIF values recorded at the previous instant. Therefore, at each step the code looked for still safe locations, where it performed the random extraction to verify the possible initiation of defect, and at the same time, when it met a cracked location, it looked for the SIF value recorded in the previous step and, considering it as constant in the step, carried out the integration of the growth law in order to obtain the new defect length.

The core of the analysis was the coupling of the code with a DBEM module, which in our case was the commercial code BEASY®; a reference input file, representing the safe structure, was prepared by the user and submitted to the code, which analyzed the file, interpreted it and defined the possible crack locations; then, after completing the evaluations needed at the particular step, it would build a new file which contained the same structure, but as damaged as it came from the current analysis and it submitted it to BEASY®; once the DBEM run was carried out, the code read the output files, extracted the SIF values pertaining to each location and performed a new evaluation. For each ligament the analysis ended when the distance between two singularities was smaller than the plastic radius, as given by Irwin

$$r_p = \frac{K_I^2}{\pi \sigma_y^2} \tag{11}$$

where σ_y is the yield stress and K_I the mode-I SIF; that measure is adopted for cracks approaching a hole or an edge, while for the case of two concurrent cracks the limit distance is considered to be given by the sum of the plastic radiuses pertaining to the two defects. Once such limit distance was reached, the ligament was considered as broken, in the sense that no larger cracks could be formed; however, to take into account the capability of the ligament to still carry some load, even in the plastic field, the same net section was still considered in the following steps, thus renouncing to take into account the plastic behaviour of the material. Therefore, the generic M-C trial was considered as ended when one of three conditions are verified, the first being the easiest, i.e. when a limit number of cycles given by the user was reached. The second possibility was that the mean longitudinal stress evaluated in the residual net section reached the yield stress of the material and the third, obviously, was met when all ligaments were broken. Several topics are to be further specified and first of all the probabilistic capabilities of the code, which are not limited to the initiation step. The extent of the probabilistic analysis can be defined by the user, but in the general case, it refers to both loading and propagation parameters.

For the latter, user inputs the statistics of the parameters, considering a joint normal density which couples lnC and n, with a normal marginal distribution for the second parameter; at each propagation step the code extracted at each location new values to be used in the integration of the growth law.

The variation of remote stress was performed in the same way, but it was of greater consequences; first of all we have to mention that a new value of remote stress was extracted at the beginning of each step from the statistical distribution that, for the time being, we considered as a normal one, and then kept constant during the whole step: therefore, variations which occurred for shorter times went unaccounted. The problem which was met when dealing with a variable load concerned the probability of crack initiation, more than the propagation phase; that's because the variation of stress implies the use of some damage accumulation algorithm, which we used in the linear form of Miner's law, being the most used one.

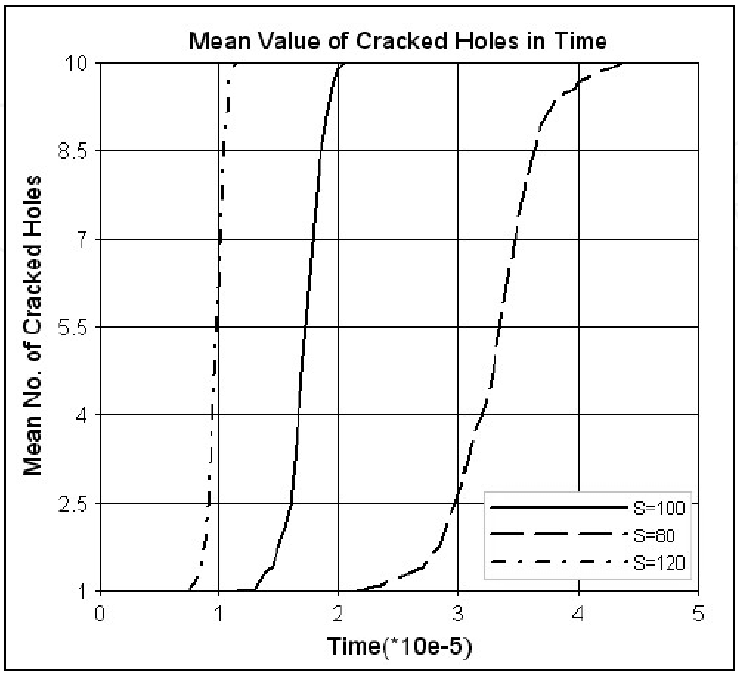


Fig. 15. Cdfs' for a given number of cracked holes in time

However, we have to observe that if the number of cycles to crack initiation is a random variable, as we considered above, the simple sum of deterministic ratios which appears in Miner's law cannot be accepted, as pointed out by Hashin (1980; 1983), the same sum having a probabilistic meaning; therefore, the sum of two random variables, i.e. the damage cumulated and the one corresponding to the next step, has to be carried out by performing the convolution of the two pdfs' involved. This task is carried out by the code, in the present version, by a rather crude technique, recording in a file both the damage cumulated at each location and the new one and then performing the integration by the trapezoidal rule.

At the end of all M-C trials, a final part of our code carried out the statistical analysis of results in such a way as to be dedicated to the kind of problem in hand and to give useful results; for example, we could obtain, as usually, the statistics of initiation and failure times, but also the cumulative density function (cdf) of particular scenarios, as that of cracks longer than a given size, or including an assigned number of holes, as it is illustrated in fig. 15.

4. Multivariate optimization of structures and design

The aim of the previous discussion was the evaluation of the probability of failure of a given structure, with assigned statistics of all the design variables involved, but that is just one of the many aspects which can be dealt within a random analysis of a structural design. In many cases, in fact, one is interested to the combined effects of input variables on some kind of answer or quality of the resulting product, which can be defined as weight, inertia, stiffness, cost, or others; sometimes one wishes to optimize one or several properties of the result, either maximizing or minimizing them, and different parameters can give to the design opposing tendencies, as it happens for example when one wishes to increase some stiffness of the designed component, while keeping its weight as low as possible.

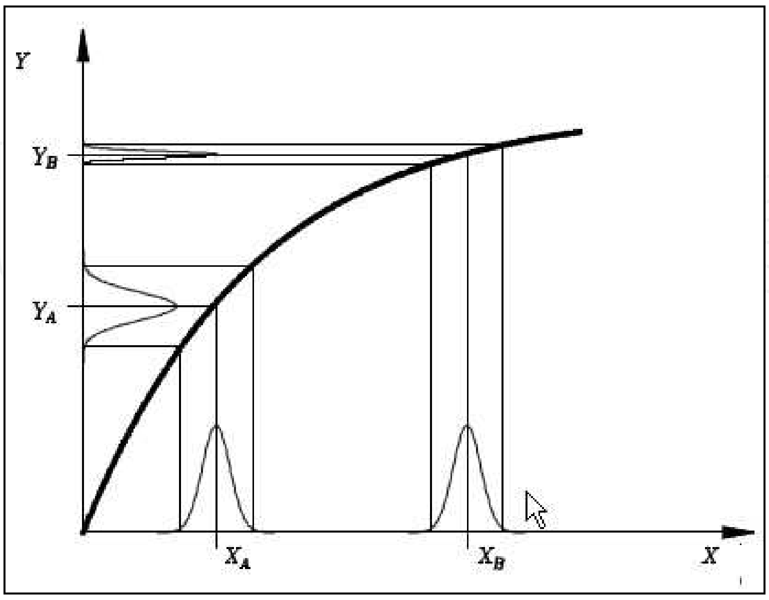


Fig. 16. How the statistics of the result depend on the mean value of the control variables

In any case, one must consider that, at least in the structural field for the case of large deformations, the relationship between the statistic of the response and that of a generic design variable for a complex structure is in general a non-linear one; it is in fact evident from fig. 16 that two different mean values for the random variable x, say x_A and x_B, even in presence of the same standard variation, correspond to responses centered in y_A and y_B, whose coefficients of variation are certainly very different from each other. In those cases, one has to expect that small variations of input can imply large differences for output characteristics, in dependence of the value around which input is centered; that aspect is of relevant importance in all those cases where one has to take into account the influences exerted by manufacturing processes and by the settings of the many input parameters (control variables), as they can give results which mismatch with the prescribed requirements, if not themselves wrong.

Two are the noteworthy cases, among others, i.e. that were one wish to obtain a given result with the largest probability, for example to limit scraps, and the other, where one wishes to obtain a design, which is called 'robust', whose sensitivity to the statistics of control variables is as little as possible.

Usually, that problem can be solved for simple cases by assigning the coefficients of variation of the design variables and looking for the corresponding mean values such as to attain the required result; the above mentioned hypothesis referring to the constancy of the coefficients of variation is usually justified with the connection between variance and quality levels of the production equipments, not to mention the effect of the nowadays probabilistic techniques, which let introduce just one unknown in correspondence of each variable.

Consequently, while in the usual probabilistic problem we are looking for the consequences on the realization of a product arising from the assumption of certain distributions of the design variables, in the theory of optimization and robust design the procedure is reversed,

as we now look for those statistical parameters of the design variables such as to produce an assigned result (target), characterized by a given probability of failure.
It must be considered, however, that no hypothesis can be introduced about the uniqueness of the result, in the sense that more than one design can exist such as to satisfy the assigned probability, and that the result depends on the starting point of the analysis, which is a well known problem also in other cases of probabilistic analysis. Therefore, the most useful way to proceed is to define the target as a function of a given design solution, for example of the result of a deterministic procedure, in order to obtain a feasible or convenient solution.
The main point of multi-objective optimization is the search for the so-called Pareto-set solutions; one starts looking for all feasible solutions, those which don't violate any constraint, and then compare them; in this way, solutions can be classified in two groups, i.e. the dominating ones, which are better than the others for all targets (the 'dominated' solutions) and which are non-dominating among each other. In other words, the Pareto-set is composed by all feasible solutions which are non-dominating each other, i.e. which are not better for at least one condition, while they are all better than the dominated solutions.
As it is clear from above, the search for Pareto-set is just a generalization of the optimization problem and therefore a procedure whatever of the many available ones can be used; for example, genetic algorithm search can be conveniently adopted, even if in a very general way (for example, MOGA, 'Multi-Objective Genetic Algorithm' and all derived kinds), coupled with some comparison technique; it is evident that this procedure can be used at first in a deterministic field, but, if we apply at each search a probabilistic sense, i.e. if we say that the obtained solution has to be a dominating one with a given probability of success (or, in reverse, of failure) we can translate the same problem in a random sense; of course, one has to take into account the large increase of solutions to be obtained in such a way as to build a statistic for each case to evaluate the required probability.
In any case, at the end of the aforesaid procedure one has a number of non-dominating solutions, among which the 'best' one is hopefully included and therefore one has to match against the problem of choosing among them. That is the subject of a 'decision making' procedure, for which several techniques exist, none of them being of general use; the basic procedure is to rank the solutions according to some principle which is formulated by the user, for example setting a 'goal' and evaluating the distance from each solution, to end choosing that whose distance is a minimum. The different commercial codes (for example, Mode-Frontier is well known among such codes) usually have some internal routines for managing decisions, where one can choose among different criteria.
More or less, the same procedure which we have just introduced can be used to obtain a design which exhibits an assigned probability of failure (i.e. of mismatching the required properties) by means of a correct choice of the mean values of the control variables. This problem can be effectively dealt with by an SDI (Stochastic Design Improvement) process, which is carried out through an convenient number of MC (here called runs) as well as of the analysis of the intermediate results. In fact, input - i.e. design variables x - and output - i.e. target y - of an engineering system can be connected by means of a functional relation of the type

$$y = F(x_1, x_2, \ldots, x_n) \tag{12}$$

which in the largest part of the applications cannot be defined analytically, but only rather ideally deduced because of the its complex nature; in practice, it can be obtained by considering a sample x_i and examining the response y_i, which can be carried out by a simulation procedure and first of all by one of M-C techniques, as recalled above. Considering a whole set of M-C samples, the output can be expressed by a linearized Taylor expansion centered about the mean values of the control variables, as

$$y_j = F(\mu_{x_i}) + \Sigma \frac{dF}{dx_i}(x_i - \mu_{x_i}) = \mu_{y_j} + \mathbf{G}\{x_i - \mu_{x_i}\} \tag{13}$$

where μ_i represents the vector of mean values of input/output variables and where the gradient matrix **G** can be obtained numerically, carrying out a multivariate regression of y on the x sets obtained by M-C sampling. If y_0 is the required target, we can find the new x_0 values inverting the relation above, i.e. by

$$x_0 = \mu_x + \mathbf{G}^{-1}\{y_0 - \mu_{y_j}\}; \tag{14}$$

as we are dealing with probabilities, the real target is the mean value of the output, which we compare with the mean value of the input, and, considering that, as we shall illustrate below, the procedure will evolve by an iterative technique, it can be stated that the relation above has to be modified as follows, considering the update between the k-th and the (k+1)-th step:

$$\mu_{x_0} = \mu_{x,k+1} = \mu_{x,k} + \mathbf{G}^{-1}(\mu_{y,k+1} - \mu_{y,k}) = \mu_{x,k} + \mathbf{G}^{-1}(\mu_{y_0} - \mu_{y,k}). \tag{15}$$

The SDI technique is based on the assumption that the cloud of points corresponding to the results obtained from a set of MC trials can be moved toward a desired position in the N-dimensional space such as to give the desired result (target) and that the amplitude of the required displacement can be forecast through a close analysis of the points which are in the same cloud (fig 17): in effects, it is assumed that the shape and size of the cloud don't change greatly if the displacement is small enough; it is therefore immediate to realize that an SDI process is composed by several sets of MC trials (runs) with intermediate estimates of the required displacement.

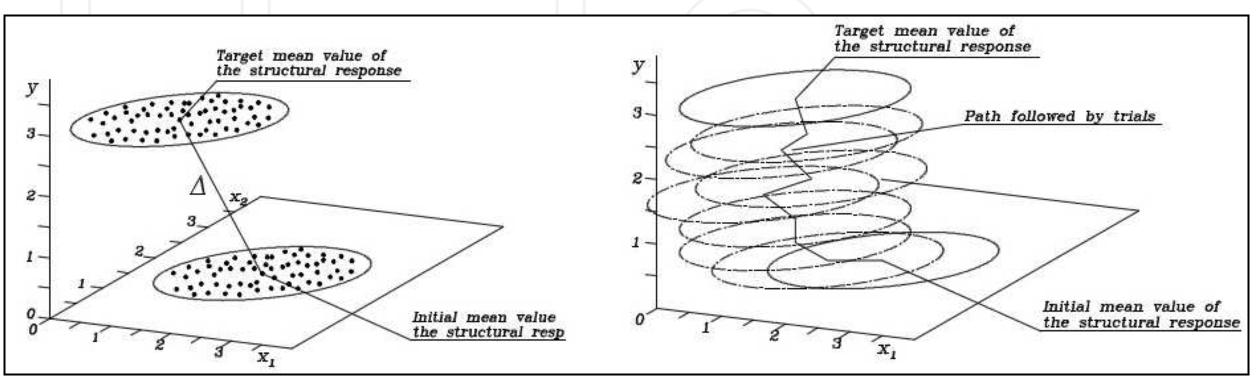


Fig. 17. The principles of SDI processes

It is also clear that the assumption about the invariance of the cloud can be kept just in order to carry out the multivariate regression which is needed to perform a new step - i.e. the evaluation of the G matrix - but that subsequently a new and correct evaluation of the cloud is needed; in order to save time, the same evaluation can be carried out every k steps, but of course, as k increases, the step amplitude has to be correspondently decreased. It is also immediate that the displacement is obtained by changing the statistics of the design variables and in particular by changing their mean (nominal) values, as in the now available version of the method all distributions are assumed to be uniform, in order to avoid the gathering of results around the mode value. It is also to be pointed out that sometimes the process fails to accomplish its task because of the existing physical limits, but in any case SDI allows to quickly appreciate the feasibility of a specific design, therefore making easier its improvement.

Of course, it may happen that other stochastic variables are present in the problem (the so called background variables): they can be characterized by any type of statistical distribution included in the code library, but they are not modified during the process. Therefore, the SDI process is quite different for example from the classical design optimization, where the designer tries to minimize a given objective function with no previous knowledge of the minimum value, at least in the step of the problem formulation. On the contrary, in the case of the SDI process, it is first stated what is the value that the objective function has to reach, i.e. its target value, according to a particular criterion which can be expressed in terms of maximum displacement, maximum stress, or other. The SDI process gives information about the possibility to reach the objective within the physical limits of the problem and determines which values the project variables must have in order to get it. In other words, the designer specifies the value that an assigned output variable has to reach and the SDI process determines those values of the project variables which ensure that the objective variable becomes equal to the target in the mean sense. Therefore, according to the requirements of the problem, the user defines a set of variables as control variables, which are then characterized from a uniform statistical distribution (natural variability) within which the procedure can let them vary, while observing the corresponding physical (engineering) limits. In the case of a single output variable, the procedure evaluates the Euclidean or Mahalanobis distance of the objective variable from the target after each trial:

$$d_i = \left| y_i - y^* \right| \qquad i = 1, 2, \ldots, N \tag{16}$$

where y_i is the value of the objective variable obtained from the i-th iteration, y^* is the target value and N is the number of trials per run. Then, it is possible to find among the worked trials that one for which the said distance gets the smallest value and subsequently the procedure redefines each project variable according to a new uniform distribution with a mean value equal to that used in such "best" trial. The limits of natural variability are accordingly moved of the same quantity of the mean in such way as to save the amplitude of the physical variability.

If the target is defined by a set of output variables, the displacement toward the condition where each one has a desired (target) value is carried out considering the distance as expressed by:

$$d_i = \sqrt{\Sigma_k \left(y_{i,k} - y_k^* \right)^2}\ , \tag{17}$$

where k represents the generic output variable. If the variables are dimensionally different it is advisable to use a normalized expression of the Euclidean distance:

$$d_i = \sqrt{\sum \omega_k \left(\delta_{i,k}\right)^2}\,, \tag{18}$$

where:

$$\delta_{i,k} = \begin{cases} \dfrac{y_{i,k}}{y_k^*} - 1, & \text{if } y_k^* \neq 0 \\ y_{i,k} & \text{if } y_k^* = 0 \end{cases} \tag{19}$$

but in this case it is of course essential to assign weight factors ω_k to define the relative importance of each variable. Several variations of the basic procedures are available; for example, it is possible to define the target by means of a function which implies an equality or even an inequality; in the latter case the distance is to be considered null if the inequality is satisfied. Once the project variables have been redefined a new run is performed and the process restarts up to the completion of the assigned number of shots. It is possible to plan a criterion of arrest in such way as to make the analysis stop when the distance from the target reaches a given value. In the most cases, it is desirable to control the state of the analysis with a real-time monitoring with the purpose to realize if a satisfactory condition has been obtained.

5. Examples of multivariate optimization

5.1 Study of a riveting operation

The first example we are to illustrate is about the study of a riveting operation; in that case we tried to maximize the residual compression load between the sheets (or, what is the same, the traction load in the stem of the rivet) while keeping the radial stress acting on the wall of the hole as low as possible; the relevant parameters adopted to work out this example are recorded in Tab. 1.

RGR	Hole Radius	Variable	mm	2.030	2.055
RSTEM	Shank Radius	Variable	mm	1.970	2.020
LGR	Shank Length	Variable	mm	7.600	8.400
AVZ	Hammer Stroke	Variable	mm	3.500	4.500
EYG	Young Modulus	Variable	MPa	65,000	75,000
THK	Sheets Thickness	Constant	mm	1.000	
SIZ	Yield Stress	Constant	MPa	215.000	
VLZ	Hammer Speed	Constant	mm/sec	250.000	

Table 1. Relevant parameters for riveting optimization

It is to be said that in this example no relevant result was obtained, because of the ranges of variation of the different parameters were very narrow, but in any case it can be useful to quote it, as it defines a procedure path which is quite general and which shows very clearly the different steps we had to follow. The commercial code used was Mode-Frontier®, which is now very often adopted in the field of multi-objective optimization; that code let the user build his own problem with a logic procedure which makes use of icons, each of them corresponding to a variable or to a step of the procedure, through which the user can readily build his problem as well as the chosen technique of solution; for example, with reference to the table above, in our case the logic tree was that illustrated in fig. 18.

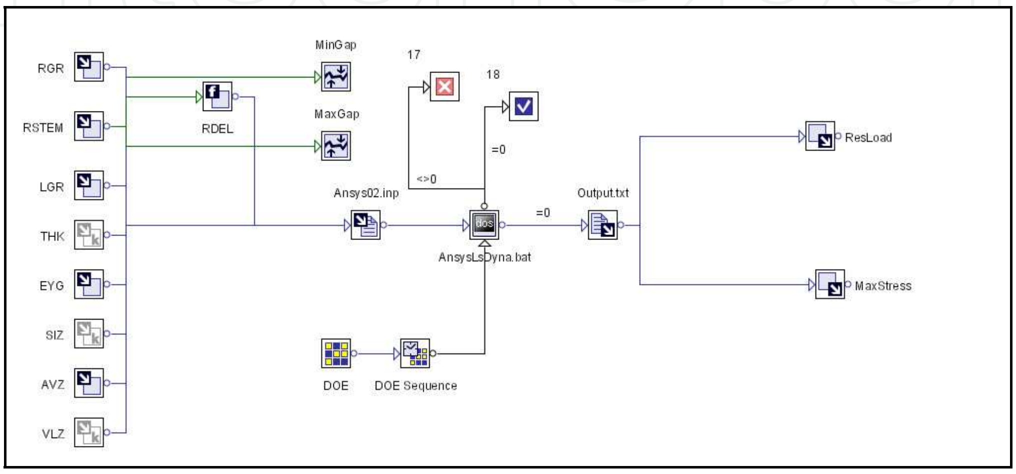


Fig. 18. The building of the problem in Mode-Frontier environment

Summarizing the procedure, after defining all variables and parameters, the work can be set to be run by means of an user-defined script (AnsysLsDyna.bat in fig. 18), in such a way that the code knows that the current values of variables and parameters are to be found somewhere (in Ansys02.inp), to be worked somehow, for example according to a DOE procedure or to a genetic algorithm or other, and that the relevant results will be saved in another file (in Output.txt in our case); those results are to be compared with all the previously obtained ones in order to get the stationary values of interest (in our case, the largest residual load and the smallest residual stress).
The kernel of the procedure, of course, is stored in the script, where the code finds how to pass from input data to output results; in our case, the input values were embedded in an input file for Ansys® preprocessor, which would built a file to be worked by Ls-Dyna® to simulate the riveting operation; as there was no correct correspondence between those two codes, a home-made routine was called to match requirements; another home-made routine would then extract the results of interest from the output files of Ls-Dyna®.
A first pass from Mode-Frontier® was thus carried out, in such a way as to perform a simple 3-levels DOE analysis of the problem; a second task which was asked from the code was to build the response surface of the problem; there was no theoretical reason to behave in such a way, but it was adopted just to spare time, as each Ls-Dyna trial was very time-expensive, if compared with the use of RS: therefore the final results were 'virtual', in the sense that they didn't came from the workout of the real problem, but from its approximate analytic representation.

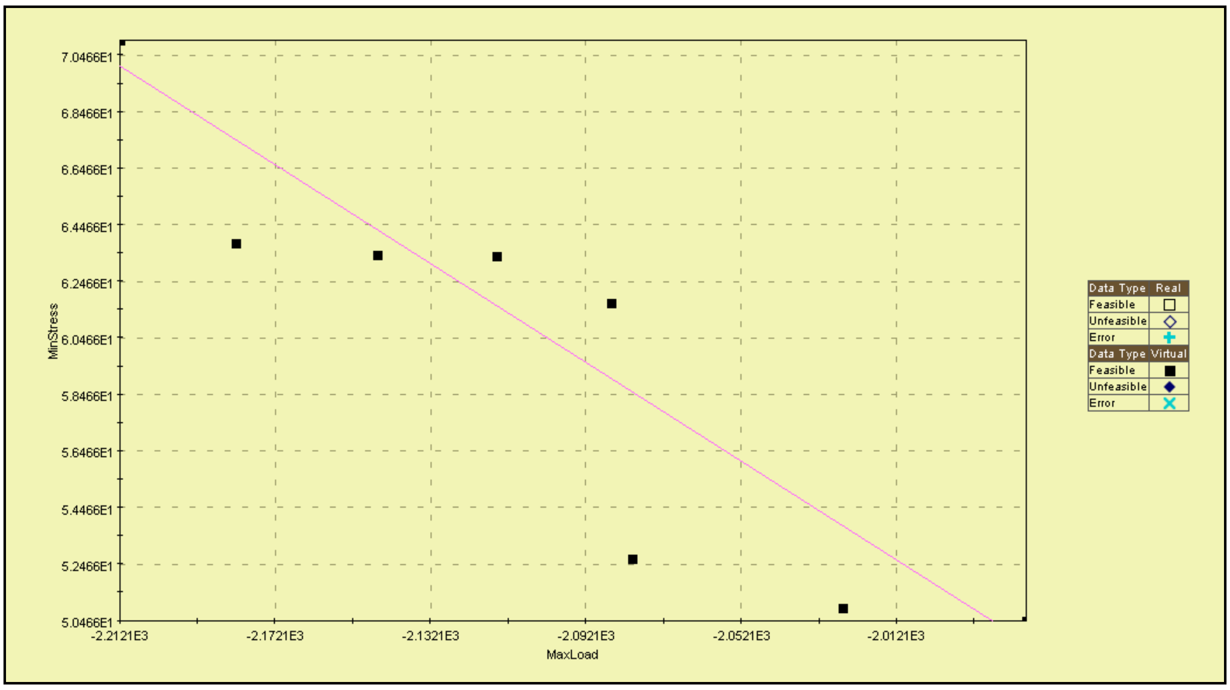


Fig. 19. Pareto-set for the riveting problem

Thus, the Pareto-set for the riveting problem was obtained, as it is shown in fig. 19; it must be realized that the number of useful non dominated results was much larger than it can be shown in the same picture, but, because of the narrow ranges of variance, they overlap and don't appear as distinct points.
The last step was the choice of the most interesting result, which was carried out by means of the Decision Manager routine, which is also a part of Mode-Frontier code.

5.2 The design improvement of a stiffened panel

As a second example we show how a home-made procedure, based on the SDI technique, was used to perform a preliminary robust design of a complex structural component; this procedure is illustrated with reference to the case of a stiffened aeronautical panel, whose residual strength in presence of cracks had to be improved. Numerical results on the reference component had been validated by using experimental results from literature.
To demonstrate the procedure described in the previous section, a stiffened panel constituted by a skin made of Al alloy 2024 T3, divided in three bays by four stiffeners made of Al alloy 7075 T5 (E = 67000 MPa, σy = 525 MPa, σu = 579 MPa, δult = 16%) was considered. The longitudinal size of the panel was 1830 mm, its transversal width 1190 mm, the stringer pitch 340 mm and the nominal thickness 1.27 mm; the stiffeners were 2.06 mm high and 45 mm wide. Each stiffener was connected to the skin by two rows of rivets 4.0 mm diameter.
A finite element model constituted by 8-noded solid elements had been previously developed and analyzed by using the WARP 3D® finite element code. The propagation of two cracks, with the initial lengths of 120 mm and 150 mm respectively, had been simulated by considering the Gurson-Tveergard model, as implemented in the same code, whose parameters were calibrated.

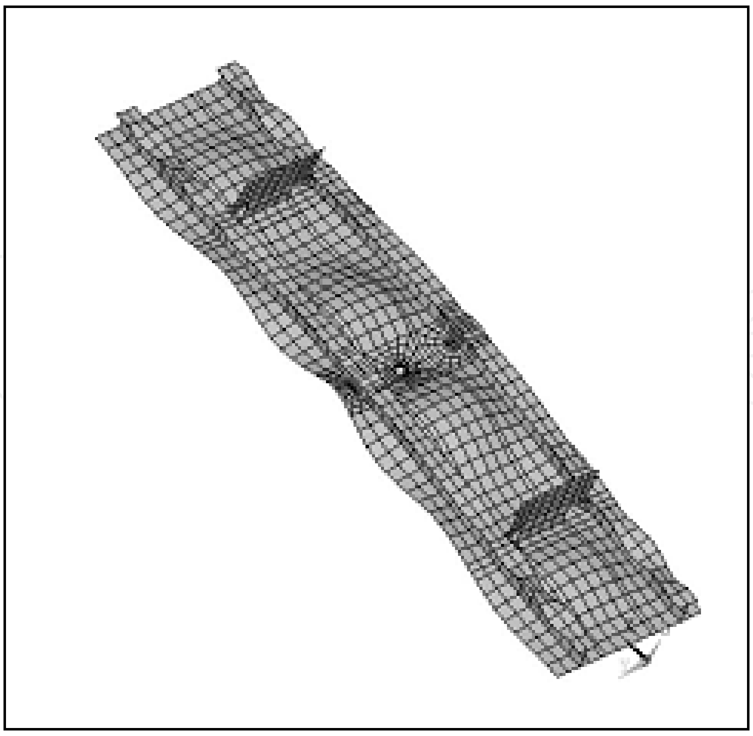


Fig. 20. The model of the stiffened panel

In the proposed application of the SDI procedure, a substructure of the panel was considered (fig. 20), corresponding to its single central bay (the part of the panel within the two central stringers) with a fixed width equal to 680 mm, where a central through-crack was assumed to exist, with an initial length of 20 mm. The pitch between the two stringers and their heights were considered as design variables. As natural variables the stringers pitch (±10.0 range) and the stringers height (±0.4 mm range) were assumed, while the engineering intervals of the variables was considered to be respectively [306 ÷374 mm] and [1.03 ÷ 3.09 mm]. An increment of the maximum value of the residual strength curve (Rmax) of the 10 %, with a success probability greater than 0.80, was assumed as the target.

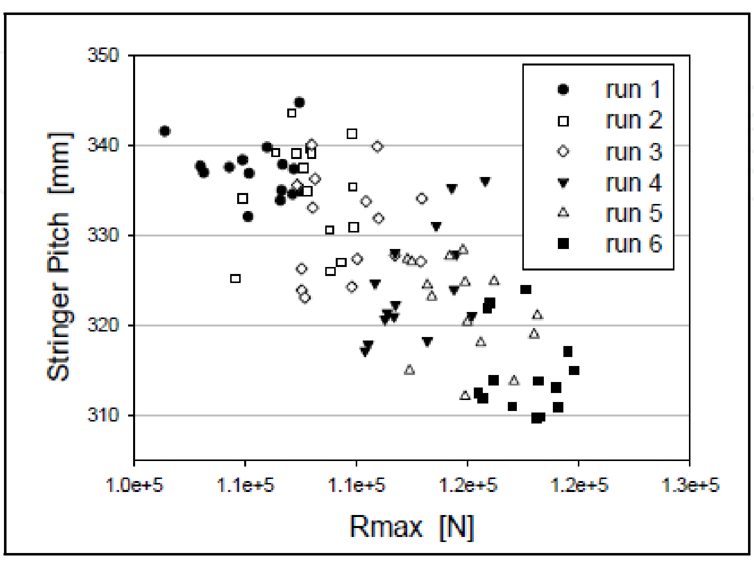


Fig. 21. Path of clouds for R_{max} as a function of the stringer pitch

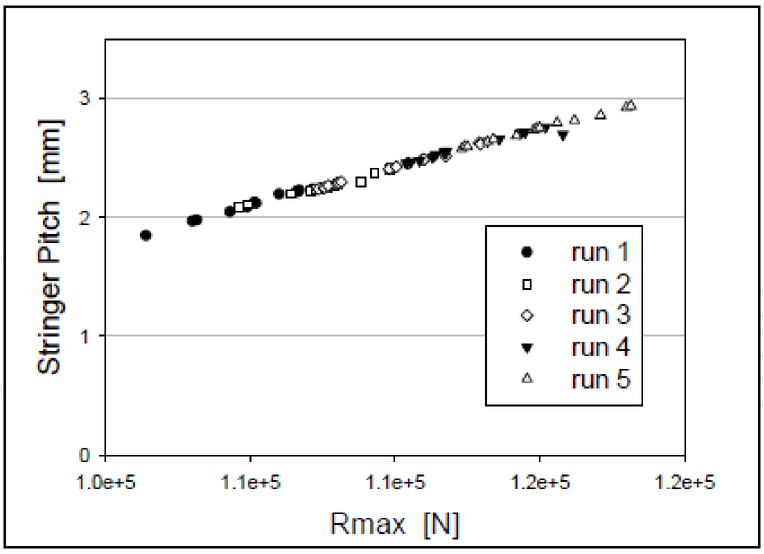


Fig. 22. Stringer pitch vs. Target

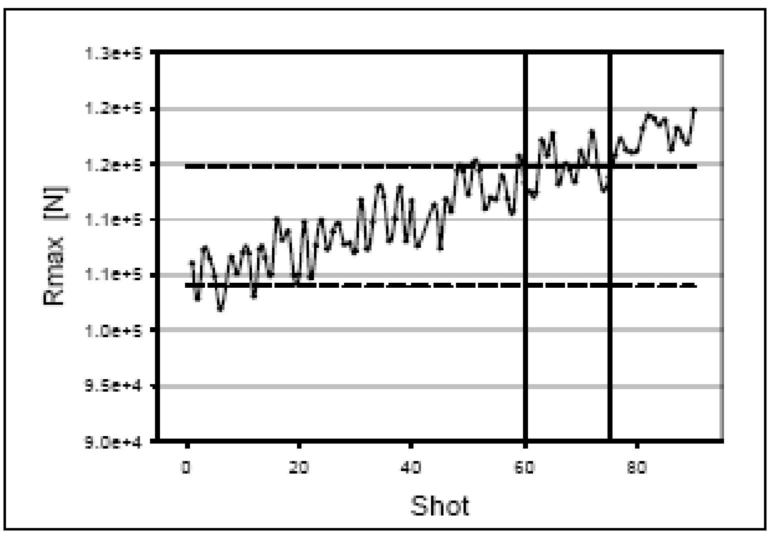


Fig 23. Target vs. shot per run

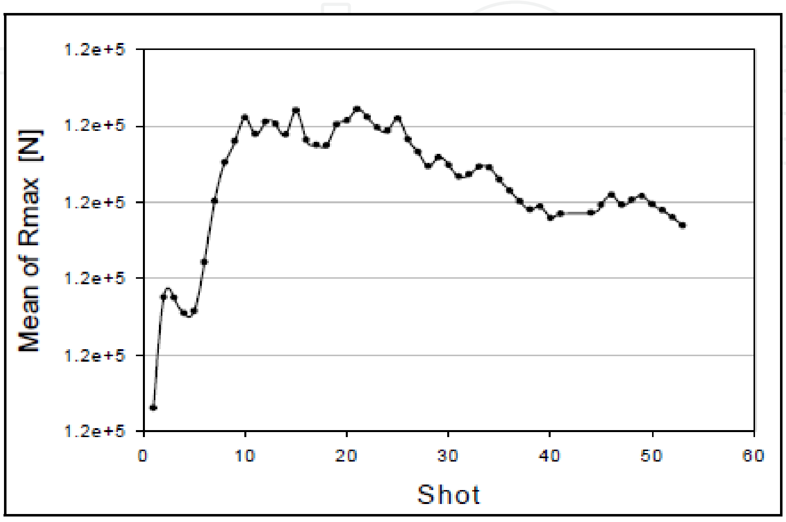


Fig. 24. Mean value of target vs. shot

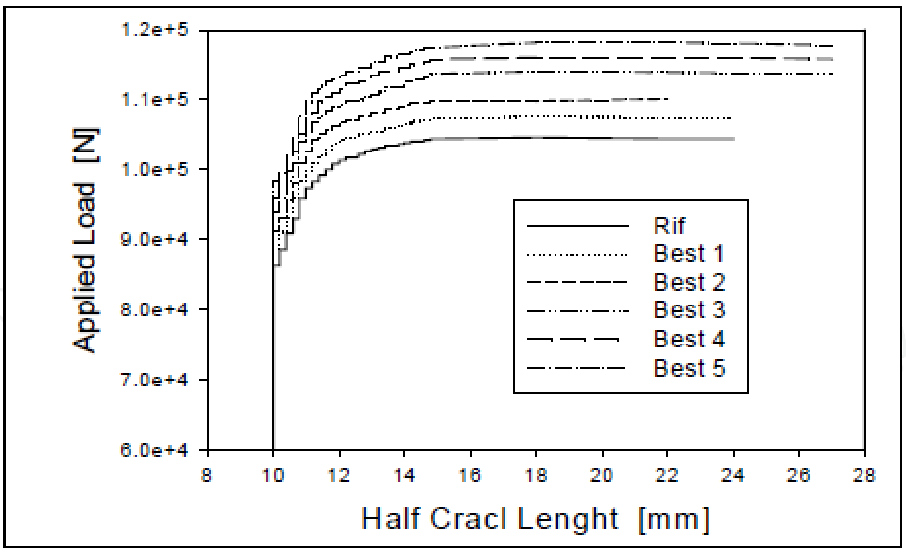


Fig. 25. R-curves obtained for the best shot of each run

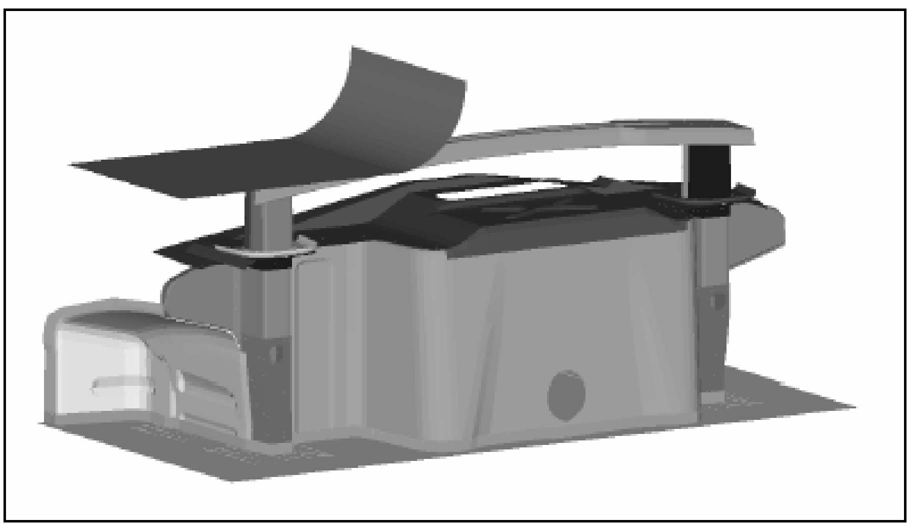


Fig. 26. The global FEM model of the absorber

A total of 6 runs, each one of 15 shots, were considered adequate to satisfy the target, even if at the end of the procedure an extended MC had to be performed in order to assess the obtained results from the 15 shots of the last satisfying run. In the following fig. 21 and 22 the design variables vs. the maximum value of the residual strength are illustrated. In correspondence to these two plots we recorded in fig. 23 the values assumed by the maximum value of the R-curve for each shot. In the same figure the reference value (obtained by considering the initial nominal value of the design variables) of the maximum value of the R-curve is reported together with the target value (dashed line). As it is possible to observe, 9 of the 15 shots of the 5th run overcame the target value; it means that by using the corresponding mean value of the design variable the probability to satisfy the target is of about 0.60.

Therefore, another run (the 6th) was carried out and just one shot didn't overcome the target value, so that the approximate probability to satisfy the target is about 0.93. The mean values of the design variables in the 6th run were respectively 318.8 mm for the stringer pitch and 2.89 mm for the stinger height; the mean value of the output variable was 116000

N. An extended MC (55 trials) was performed on the basis of the statistics of the 6th run and the results showed in the fig. 24 were obtained, where the mean value of the residual strength vs. the number of the trial has been recorded. The new mean of the output variable was 117000 N with a standard deviation of 1800 N and the probability to satisfy the target was exactly 0.80. At the end, in fig. 25, the six R-curves corresponding to the six best shots for each run are reported, together with the reference R-curve.

5.3 Optimization of an impact absorber

As a last example, the SDI procedure was applied to reduce the maximum value of the displacement in time of a rigid barrier that impacted the rear substructure of a vehicle (fig. 26) in a crash test. The reasons which lie behind such a choice are to be found in the increasing interest in numerical analysis of crashworthiness of vehicles because of the more strict regulations concerning the protection of the occupants and related fields. In Europe the present standards to be used in homologation of vehicles are more or less derived by U.S. Code of Federal Regulations, CFR-49.571, but ever-increasing applications are done with reference to other standards, and first of all to EURONCAP. The use of such standards, who are mainly directed to limit biomechanical consequences of the impact - which are controlled by referring the results to standard indexes related to different parts of human body - implies that, besides the introduction of specific safety appliances, as safety belts and airbags, the main strength of car body has to be located in the cell which holds passengers, in order to obtain a sufficient survival volume for the occupants; the other parts of the vehicle are only subsidiary ones, because of the presence of absorbers which reduce the impact energy which is released on the cell.

We can add to all previous considerations that the present case study was adopted as it is well known that vehicle components come from mass production, where geometrical imperfections are to be expected as well as a certain scatter of the mechanical properties of the used materials; therefore, it can't be avoided that the said variations induce some differences of response among otherwise similar components, what can be relevant in particular cases and first of all in impact conditions; the analysis which we carried out was directed to define, through the use of the previously sketched procedure, the general criteria and the methodology required to develop a robust design of those vehicle components which are directly used to limit plastic deformations in impact (impact absorber). In our particular case, the study was carried out with reference to the mentioned substructure, whose effective behaviour in impact (hammer) conditions is known and is associated to those deterministic nominal values of the design variable actually in use, with the immediate objective to obtain a reduction of the longitudinal deformation of the impact absorber.

The substructure is a part of a rear frame of a vehicle, complete with cross-bar and girders, where impact absorbers are inserted; the group is acted upon by a hammer which is constrained to move in the longitudinal direction of the vehicle with an initial impact speed of 16 km/h; the FE model used for the structure consisted of about 23400 nodes and about 21900 elements of beam, shell and contact type, while the hammer was modelled as a “rigid wall". The thicknesses of the internal and external C-shaped plates of the impact absorbers were selected as project variables, with a uniform statistical distribution in the interval [1.7mm±1.9mm]; lower and upper engineering limits were respectively 1.5 mm and 2.1 mm.

This choice was carried out by preliminary performing, by using the probabilistic module of ANSYS® ver 8.0 linked to the explicit FE module of LS-Dyna® included in the same code, a sensitivity analysis of an opportune set of design variables on the objective variable, which is, as already stated before, the maximum displacement of the hammer.
As design variables to be involved in the sensitivity analysis we chose, besides the inner and outer thicknesses of the C-shaped profile of the impact absorbers, the mechanical properties of the three materials constituting the main components of the substructure (the unique young modulus and the three yielding stresses); it was clear from the obtained results that while the relationship existing between the thicknesses of the inner and outer C-shaped profile of the impact absorber and the objective variable is quite linear, as well as the relationship between the yielding stress of the impact absorber material and the same objective variable, a relationship between the other considered variables and the objective variable is undetermined.

Variable	Type	Distribution	Natural variability	Physical limits
Internal plate thick.	Design var.	uniform	1.7-1.9	1.5-2.1
External plate thick.	Design var.	uniform	1.7-1.9	1.5-2.1
material scale factor DC04 strain-rate 0	independent var.	uniform	0.95-1.05	
material scale factor DC04 strain-rate 0.005	dependent var.			
material scale factor DC04 strain-rate 0.05	dependent var.			
material scale factor DC04 strain-rate 0.5	dependent var.			
material scale factor DP600 strain-rate 0	independent var.	uniform	0.95-1.05	
material scale factor DP600 strain-rate 0.005	dependent var.			
material scale factor DP600 strain-rate 0.05	dependent var.			
material scale factor DP600 strain-rate 0.5	dependent var.			
material scale factor S355NC strain-rate 0	independent var.	uniform	0.95-1.05	
material scale factor S355NC strain-rate 0.03	dependent var.			
material scale factor S355NC strain-rate 0.5	dependent var.			

Table 2. The properties of the variables used in the absorber case

It was also quite evident that the only design variables which influence the objective one were the mechanical properties of the material of the impact absorber and the thicknesses of its profiles. A preliminary deterministic run, carried out with the actual design data of the structure gave for the objective variable a 95.94 mm “nominal" value, which was reached after a time of 38.6 ms from the beginning of the impact. The purpose of SDI in our case was

assumed the reduction of that displacement by 10% with respect to this nominal value and therefore an 86.35 mm target value was assigned.
The mechanical properties of the three materials constituting the absorbers and the rear crossbar of the examined substructure were also considered as random; it was assumed that their characteristic stress-strain curves could vary according to a uniform law within 0.5% of the nominal value. This was made possible by introducing a scale factor for the characteristic curves of the materials, which were considered as uniformly distributed in the interval [0.95,1.05].
Moreover, four stress-strain curves were considered for each material, corresponding to as many specific values of the strain-rate. The relationship among those curves and the static one was represented, according to the method used in Ls-Dyna®, by means of a scale factor which let us pass from one curve to another as a function of the strain-rate; also those factors were assumed to be dependent on that applied to the static curve, in order to avoid possible overlapping.
Therefore, the simulation involved 14 random variables, among which only 2 were considered as project variables; in the following Tab. 2 the properties of all the variables are listed. To work out the present case, the commercial well known St-Orm® code was used coupled with the Ls-Dyna® explicit solver for each deterministic FEM analysis and ran on a 2600 MHz bi-processor PC, equipped with a 2 Gb RAM; SDI processing required 9 runs with 25 shots each, with a total of 225 MC trials, and the time required to complete a single LS-Dyna® simulation being of about 2 hours.
As we already pointed out, the stochastic procedure develops through an MC iterative process where the input variables are redefined in each trial in such a way as to move the results toward an assigned target; therefore, we need first to assess what we mean as an attained target.
After every run the statistics of the output variables could be obtained, as well as the number of times that the target was reached, which could be considered as the probability of attainment of the target for the particular design, expressed through the mean values of the input variables. It is noteworthy to specify that these data are only indicative, because the MC procedure developed within a single set of trials is not able to give convergent results due to the low number of iterations.
Therefore, considering the procedure as ended when a run was obtained where all trials gave the desired target value, it was opportune to perform a final MC convergent process to evaluate the extent by which the target had been indeed reached, using the statistical distributions of the variables of the last run. For the same reason, a real-time monitoring could induce the designer to stop the procedure even if not all trials - but “almost" all - of the same run give the target as reached, also to comply with production standard and procedures. As we already pointed out in the previous paragraphs, the stochastic procedure develops through an MC iterative process where the input variables are redefined in each trial in such a way as to move the results toward an assigned target; therefore, we needed first to assess what we meant as an attained target.
For what concerns our case study, the detailed data for every run are recorded in Tab. 3; if compared with the first run, the mean of the displacement distribution in the 9th run is reduced of 8.6% and 23/25 shots respect the target: therefore, the results of the 9th run may be considered as acceptable.

	Thickness of the internal sheet [mm]			Thickness of the external sheet [mm]			Displacement [mm]		Distance from target	
run	left bound	mean value	right bound	left bound	mean value	right bound	mean value	std	mean value	std
1	1.7000	1.8000	1.9000	1.7000	1.8000	1.9000	95.9524	2.4504	0.0585	0.0270
2	1.7903	1.8903	1.9903	1.7909	1.8909	1.9909	91.5568	2.3917	0.0158	0.0190
3	1.7127	1.8127	1.9127	1.8322	1.9322	2.0322	92.5833	2.4903	0.0240	0.0232
4	1.7297	1.8297	1.9297	1.8795	1.9795	2.0795	91.0362	2.3111	0.0132	0.0144
5	1.7624	1.8624	1.9624	1.9475	2.0238	2.1000	88.8615	2.1464	0.0026	0.0061
6	1.7632	1.8632	1.9632	2.0000	2.0500	2.1000	88.0821	1.6937	0.0005	0.0016
7	1.7446	1.8446	1.9447	1.9259	2.01295	2.1	89.6456	2.2662	0.005384	0.01072
8	1.7831	1.8831	1.9831	1.9385	2.0193	2.1	88.5974	2.1700	0.002176	0.00664
9	1.8778	1.9778	2.0778	**1.8846**	**1.9846**	**2.0846**	87.6936	2.1465	0.0009655	0.00367

Table 3. Values of control variables in the different runs

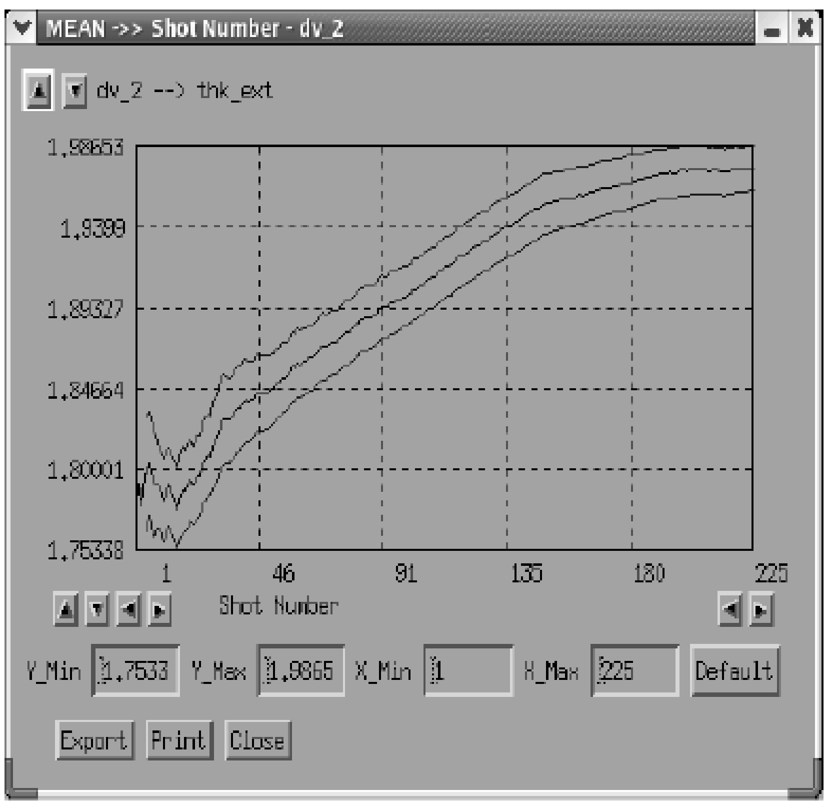


Fig. 27. Thickness of internal plate vs. shot

In the plots of fig. 27 and fig. 28 the values of the thickness of the internal and external plates of the impact absorber versus the current number of shots have been illustrated. The variable which was subjected to the largest modifications in the SDI procedure was the thickness of the external plate of the impact absorber and in fact from an analysis of sensitivity it resulted to influence at the largest extent the distance from the target. For what concerns the other random variables, it resulted from the same analysis of sensitivity that only the material of the absorbers influences in a relevant measure the behaviour of the substructure.

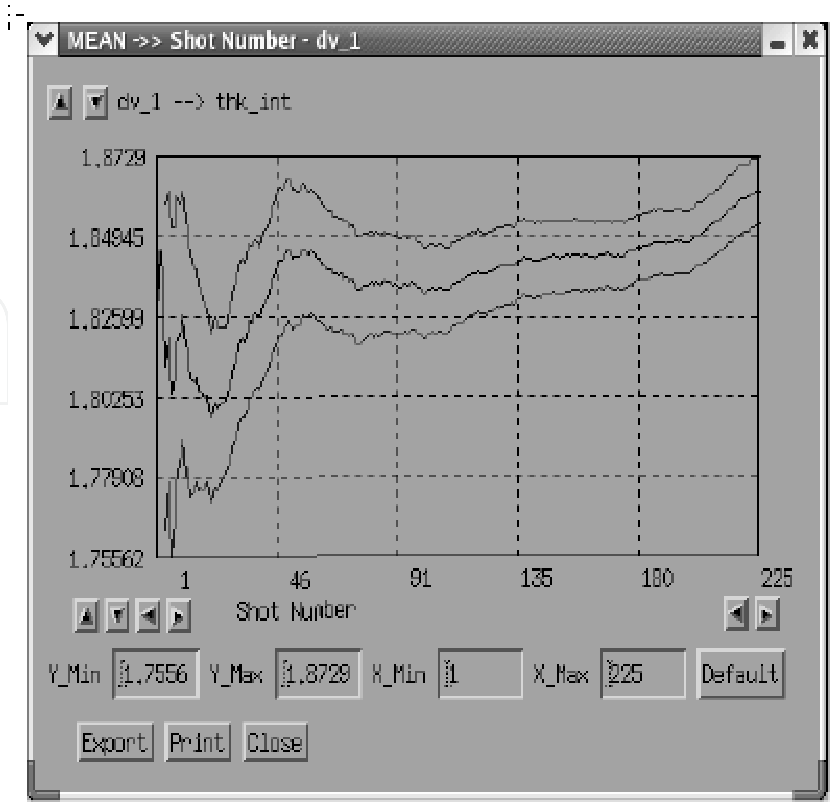


Fig. 28. Thickness of external plate vs. shot

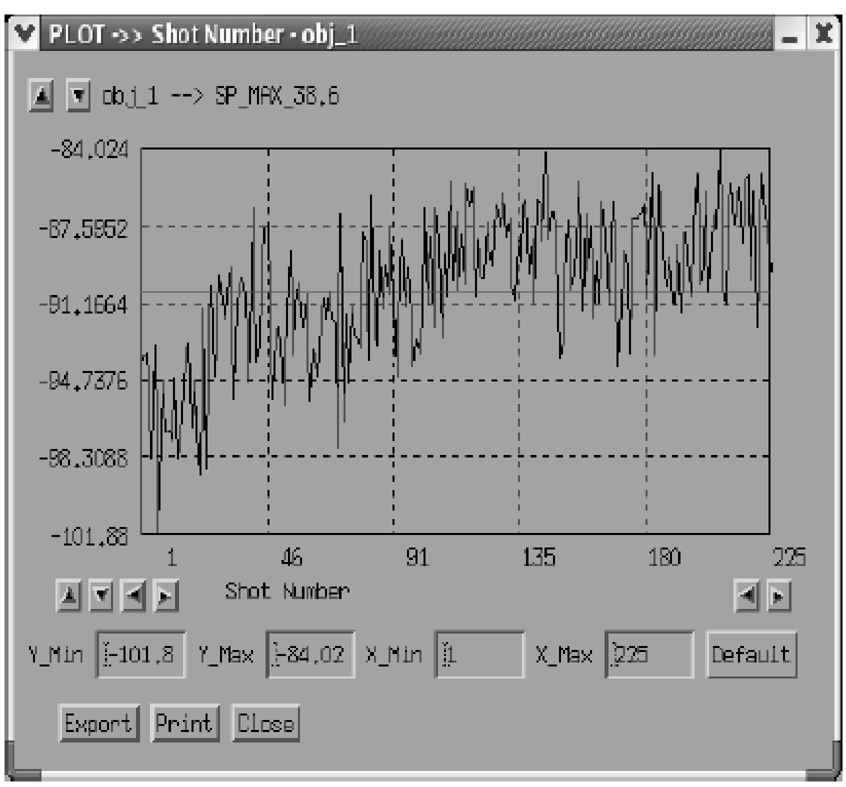


Fig. 29. Scatter plot of the objective variable

It is possible to appreciate from the scatter plots of fig. 29 and fig. 30 how the output variable approached the target value: in the 9th run, only 2 points fall under the line that represents the target and in both cases the distance is less than 0.02 and that is why the 9th run has been considered a good one, in order to save more iterations. In fig. 31 the experimental data related to the displacement of the rigid barrier vs. the time are recorded together with the numerical results obtained before and after the application of the SDI procedure.

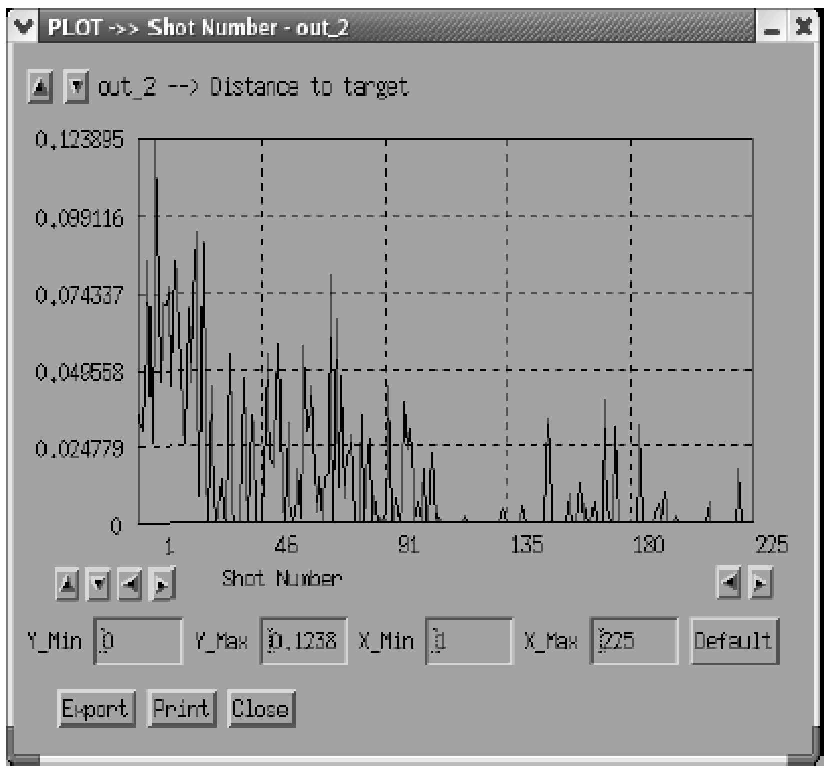


Fig. 30. Scatter plot of the distance to target

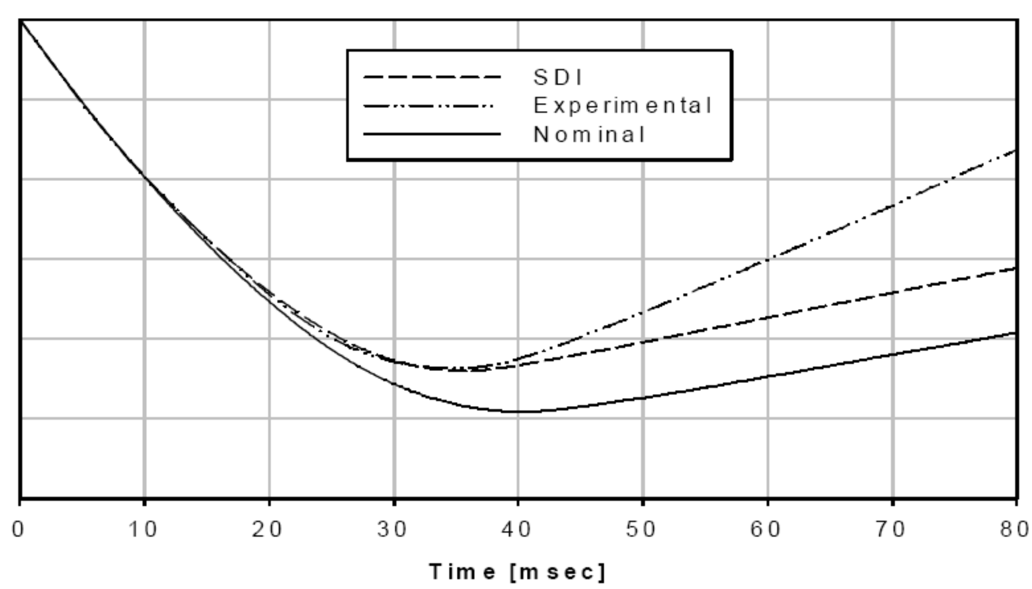


Fig. 31. Output variable vs. Time

The new nominal value of the design variables after the application of SDI procedure is 1.98 mm for both of them. A very good agreement of the numerical solution is observed in comparison to the experimental data in the first part of the curve, where they are practically overlapped and where the attention has been focused during the development of numerical simulations necessary to complete the SDI process. The general conclusion from this study was that the classical numerical simulations based on nominal values of the input variables are not exhaustive of the phenomenon in the case of crash analyses and can bring to incorrect interpretations of the dynamic behaviour of the examined structure. On the contrary, by using an SDI approach, it is possible to have a better understanding of the influence of each input variable on the structural dynamic behaviour and to assign the most appropriate nominal values in order to have results as near as possible to the target values, also in presence of their natural variability.

6. Some useful commercial codes

To fully appreciate the examples above, it may be interesting to summarize briefly the main characteristics of the commercial codes we mentioned in the preceding sections and which have interesting capabilities in the probabilistic analysis of structures; what follows doesn't want to constitute either a complete listing or an assessment of value for those codes, but only a survey of the codes we have used insofar, here published to clarify some of the topics we have just described.

First of all we have to recall that the recent versions of Ansys® couple the well-established deterministic capabilities in FE field with some new routines which work in a probabilistic environment; that innovation is so much interesting because, as we already pointed out, the design refers to structures whose study can't be carried out in a closed form by recourse to an analytical formulation; in those cases we can only hope to obtain the answer of the structure for a given set of loads and boundary conditions and therefore an FE run corresponds just to a single value of the variable set. It is only natural, therefore, that Ansys® extended its capabilities to carry out a Monte-Carlo analysis of the problem, for given statistics of the different variables and parameters.

Therefore, on the basis of a sample file (which Ansys® calls the "analysis file" of the problem) using the well renowned capabilities of its pre-processor, it is possible to characterize each variable with a distribution – to be chosen among a rather limited family of types – and then entrust the code with the task to perform a given amount of trials, from whose results the statistics of the response, as well as its sensitivities, can be recovered. A very useful characteristic of Ansys® is that the code can build a Response Surface on the basis of the obtained results and can carry on new trials using it; therefore it is quite common to carry out a limited number of M-C trials – whose amount depends on the complexity of the structure – maybe using some DOE choice, by which the Response Surface can be derived.

NESSUS®, distributed by SWRI, is a widely known and fully probabilistic code; it includes several probabilistic distributions to represent parameters and variables and is provided with both analytical and simulative methods; even if FORM, SORM, AMV+ and others are present, its main feature is the capability to be interfaced with Ansys®, Abaqus®, Ls-Dyna®, other FE codes and, at last, even with Matlab®, which widens the range of problems it can deal with; under those circumstances, it can work not just with the basic M-C method, but also with a complete set of numerical simulative methods such as Importance Sampling, Adaptive Importance Sampling, and others. The outputs it can give are such as the cumulative distribution function of the results, the probability of failure or the performance level for a given probability of failure, the probabilistic sensitivity factors and the confidence bounds of the requested result. One important feature of NESSUS® is its capability to deal not only with components but also with systems, via such methods as the Efficient Global Reliability Analysis and the Probabilistic Fault-Tree Analysis.

STRUREL®, distributed by RCP, is a complete package which is similar to NESSUS®, but has many more capabilities, as it can deal, for example, with both time-invariant and time-variant problems. It is really much more difficult to be used, but its advantages are quite evident for the expert user; beside the capabilities we already quoted for the previous code, it can carry out risk and cost analysis, failure mode assessment, reliability assessment for damaged structure, development and optimisation of strategies for inspection and maintenance, reliability oriented structural optimisation. It can also be interfaced with Permas® FE code and with user-made Fortran® routines, in such a way as to make the user able to match with very general and complex problems; a last, but very important feature is the capability to carry out random vibration analysis, with reference, for example, to wave, wind and earthquake loading.

The next two codes are of quite different nature, as they are to be used when one is interested in optimisation and in the building of a robust design. The first one, ST-Orm®, distributed by EASi Engineering, uses the SDI technique to find the setting of the control variables of a design which ensures that the assigned target is reached with a given probability; it uses M-C to obtain a cloud of results and then, applying multilinear regressions and new M-C trials, it generates families of new clouds to reach the desired target value. It claims to be a meta-code, in the sense that its tasks can be subdivided among a number of computers, each one performing a simple task in parallel, in order to save time. An useful characteristic of this code is the possibility to distinguish among the control variables, which are probabilistic variables which can vary in each cloud, and environment parameters which, even if random in character, always exhibit the same distribution, i.e. they are not displaced with clouds. All variables and parameters span in their ranges, which can vary with clouds but cannot go beyond the physical limits which are given by the user, in such a way as to exclude impossible runs.

The last code is the well assessed Mode-Frontier®, whose aim is to carry out a multi-objective optimisation for a given problem; it works with both deterministic and random variables and one of its capabilities is to build the logic of the problem by means of an iconic and very appealing method; as we already discussed, the kernel of the code is formed by a script which can be used to organize all operations and to interface with a large number of external routines. Once the Pareto-set of the problem is obtained, it can be submitted to the Decision Manager of the code, which, following different methods can help the user to choose among the previous results the one which is more convenient.

7. Conclusions and acknowledgments

From all the preceding sections it is apparent how a probabilistic study can contribute to the improvement of a structural design, as it can take into account the uncertainties that are present in all human projects, getting to such an accurate result as to examine also the

manufacturing tolerances and coming to optimize scraps. It is to be understood, however, that such results are not easy to obtain and that it seldom happens that a first-trial analysis is a sound one: a good result is only achieved after many steps have been carried out, and first of all an accurate calibration with experimental data. It happens indeed that one of the more thorny problems the user has to struggle with is the accurate description of the material used in a particular application, as new problems usually require the description of the behaviour of the material in very particular conditions, which is not often available; therefore it happens that new tests have to be created in order to deal with new specifications and that the available tests only apparently match the requirements.
It is quite clear, therefore, that in many cases the use of such new techniques can be justified only in particular conditions, for example when one is dealing with mass production, or when failure involves the loss of many lives or in other similar conditions.
We want to acknowledge the help given by the technicians of many firms, as well by the researchers of our University, first of all by ing. G. Lamanna, to cooperate – and sometimes also to support – the researches which have been quoted in the present chapter.

8. References

Boyd-Lee, A.D., Harrison G.F. & Henderson, M.B. (2001). Evaluation of standard life assessment procedures and life extension methodologies for fracture-critical components, *International Journal of Fatigue,* vol. 23, Supplement no. 1, pp. 11-19, ISSN: 0142-1123

Caputo., F., Soprano, A. & Monacelli, G. (2006). Stochastic design improvement o fan impact absorber, *Latin American Journal of Solids and Structures,* vol. 3, no. 1, pp. 41-58, ISSN: 1679-7817

Deb,K. (2004). *Multi-Objective Optimization using Evolutionary Algorithms,* John Wiley & Sons, ISBN: 0-471-87339-X, Chichester, West Sussex, UK

Deodatis, G., Asada., H. & Ito., S. (1996). Reliability of aircraft structures under non-periodic inspection: a bayesian approach, *Engineering Fracture Mechanics,* vol. 53, no. 5, pp. 789-805, ISSN: 0013-7944

Doltsinis, I., Rau, F. & Werner., M. (1999). Analysis of random systems, in: *Stochastic Analysis of Multivariate Systems in Computational Mechanics and Engineering,* Doltsinis, I. (Ed.), pp. 9-159, CIMNE-International Center for Numerical Methods in Engineering, ISBN: 84-89925-50-X, Barcelona, Spain

Du Bois, P.A. (2004). *Crashworthiness Engineering Course Notes,* Livermore Software Technology Corp., ISBN: ………………, Livermore, Ca. USA

Horst, P. (2005). Criteria for the assessment of multiple site damage in ageing aircraft. *Structural Integrity & Durability,* vol. 1, no. 1, pp. 49-65, ISSN: 1551-3750

Langrand, B.; Deletombe, E.; Markiewicz, E. & Drazétic., P. (1999). Numerical approach for assessment of dynamic strength for riveted joints. *Aerospace Science & technology*, vol. 3, no.7, pp. 431-446, ISSN: 1270-9638

Langrand, B.; Patronelli, L.; Deletombe, E.; Markiewicz, E. & Drazétic., P. (2002). An alternative numerical approach for full scale characterization for riveted joint design, *Aerospace Science & Technology,* vol. 6, no.5, pp. 345-354, ISSN: 1270-9638

Melchers, R.E. (1999). *Structural reliability analysis and prediction,* John Wiley & Sons, ISBN: 0-471-98771-9, Chichester, West Sussex, UK

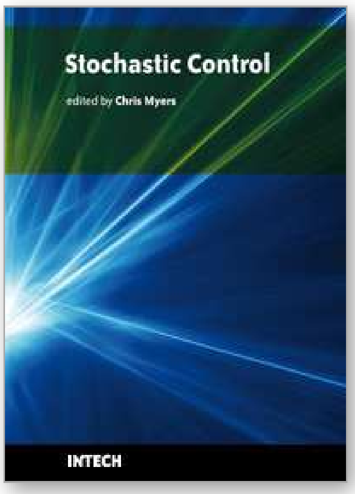
Murphy, T.E., Tsui, K.L. & Allen, J.K. (2005). A review of robust design for multiple responses, *Research in Engineering Design*, vol. 16, no. 3, pp. 118-132, ISSN: 0934-9839

Soprano, A. & Caputo., F. (2006). Building risk assessment procedures, *Structural Durability & Health Monitoring,* vol. 2, no. 1, pp. 51-68, ISSN: 1930-2983

Tani, I.; Lenoir, D. & Jezequel, L. (2005). Effect of junction stiffness degradation due to fatigue damage ofmetallic structures. *Engineering Structures,* vol. 27, no. 11, pp. 1677-1688, ISSN: 0141-0296

Urban, M.R. (2003). Analysis of the fatigue life of riveted sheet metal helicopter airframe joints, *International Journal of Fatigue,* vol. 25, no. 9-11, pp. 1013-1026, ISSN: 0142-1123

Zang, C. Friswell., M.I. & Mottershead, J.E. (2005). A review of robust optimal design and its application in dynamics, *Computers and Structures,* vol. 83, no. 4-5, pp. 315-326, ISSN: 0045-799



Stochastic Control
Edited by Chris Myers

ISBN 978-953-307-121-3
Hard cover, 650 pages
Publisher Sciyo
Published online 17, August, 2010
Published in print edition August, 2010

Uncertainty presents significant challenges in the reasoning about and controlling of complex dynamical systems. To address this challenge, numerous researchers are developing improved methods for stochastic analysis. This book presents a diverse collection of some of the latest research in this important area. In particular, this book gives an overview of some of the theoretical methods and tools for stochastic analysis, and it presents the applications of these methods to problems in systems theory, science, and economics.