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**P·SPR·D and P·SPR·D+I Control of  
Robot Manipulators  
and Redundant Manipulators**

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**1. Introduction**

This chapter is concerned with a PID-like control scheme for robot manipulators. We propose P·SPR·D control and P·SPR·D+I control for a set-point servo problem of the robot manipulators which are passive systems. P·SPR·D control consists of Proportional (P) action + Strict Positive Real (SPR) action + Derivative (D) action. Such control can asymptotically stabilize multi-joint robot manipulators. Stability analysis of the P·SPR·D control is made, based on the passivity theory and LaSalle’s invariance principle. The  $L_2$ -gain disturbance attenuation problem is also investigated. The effectiveness of the proposed method is demonstrated by the simulation results for a two-link manipulator. Let  $u \in R^r$  be the control input,  $y \in R^m$  the output,  $r \in R^m$  the desired value and  $e = r - y$  the error, then PID control is expressed as follows.

$$u = K_P e + K_I \int_0^t e(\tau) d\tau + K_D \dot{e} + m_0$$

i.e.

$$\dot{z} = e, \quad z(0) = 0$$
$$u = K_P e + K_I z + K_D \dot{e} + m_0$$

where  $K_P, K_I, K_D \in R^{r \times m}$  are gain matrices corresponding to Proportional, Integral and Derivative action, respectively, and  $m_0$  denotes the so-called manual reset quantity. We propose the following P·SPR·D control (Shimizu, 2009a) in which a SPR (strict positive real) element is used instead of Integral element:

$$\dot{\xi} = D\xi + e + \dot{e}, \quad \xi(0) = 0, \quad D < 0$$
$$u = K_P e + K_S \xi + K_D \dot{e} + m_0$$

In Section 2 we study stability analysis of the P·SPR·D control imitating the PID control for a set-point servo problem of multi-joint manipulator systems. In regard to PD and PID control for robot manipulators, there exist many papers including (Arimoto, 1996; Arimoto & Miyazaki, 1984; Spong et al., 1992), etc.. So the feature of our method is to apply the P·SPR·D control instead of PID. By introducing the SPR element it has a merit that a design of passivity-based control becomes very simplified.

When the P·SPR·D control is applied to a plant possessing the Kalman-Yakubovich-Popov (K-Y-P) property (Byrnes et al., 1991), we can prove that the closed-loop system becomes asymptotically stable by the P·SPR·D control, applying the passivity theory and LaSalle's theorem (LaSalle & Lefschetz, 1961). The reader may refer to (Shimizu, 2008a;b; 2009b) with respect to the researches of P·SPR·D control for affine nonlinear systems and mechanical systems. SPR stabilization of mechanical system is discussed in (Lozano et al., 2000) also. The reader may refer to (Byrnes et al., 1991; Khalil, 2002; Lozano et al., 2000; van der Schaft, 2000) about passivity-based control theory in general.

By the way, static state feedback control law may be obtained by the passivity based design (Sepulchre et al., 1997; Shen, 2004) of the cascaded system also. Generally speaking, however, the control law using a storage function is complex. Besides, an advantage of the P·SPR·D control is of output feedback of simple structure.

In Section 3 an extension to redundant manipulators is investigated. Control of the redundant manipulator in the task space was studied in (Arimoto, 1996; Galicki, 2008; Khatib, 1987; Murakami et al., 2008; Shibata, 2007; Spong et al., 1992).

Section 4 investigates  $L_2$ -gain disturbance attenuation problem ( $\gamma$ -dissipativity (van der Schaft, 2000)) under the existence of disturbances. It is easy to solve the problem by applying the P·SPR·D control.

The simulation results is presented in Section 5 to demonstrate the effectiveness of the proposed methods.

## 2. P·SPR·D Control of Robot Manipulators

We consider a set-point servo problem for robot manipulators. An equation of motion of robot manipulator with  $n$  joints can be obtained by the Euler-Lagrange formulation. Let  $q$  be the position (angles of each link) of the manipulator,  $\tau$  the input torque,  $\frac{1}{2}\dot{q}^T M(q)\dot{q}$  the kinetic energy and  $U(q)$  the potential energy. Then it can be represented as (Arimoto, 1996; Spong et al., 1992)

$$M(q)\ddot{q} + \frac{1}{2}\dot{M}(q)\dot{q} + S(q, \dot{q})\dot{q} + g(q) = \tau \quad (1)$$

where  $M(q)$  denotes the inertia matrix which is positive definite and bounded,  $g(q) \triangleq U_q(q)^T$  is the gradient of the gravity potential energy and  $S(q, \dot{q})$  denotes

$$S(q, \dot{q})\dot{q} = \frac{1}{2} \left\{ \dot{M}(q)\dot{q} - \left[ \frac{\partial}{\partial q} q^T M(q) \dot{q} \right]^T \right\}$$

, which is a skew-symmetric matrix. Denoting  $x_1 = q \in R^n$ ,  $x_2 = \dot{q} \in R^n$ ,  $x = (x_1^T, x_2^T)^T$ , and letting the output by  $y = x_2 \in R^n$ , and the control input by  $\tau \in R^n$ , state space representation of (1) becomes as follows.

$$\dot{x}_1 = x_2 \quad (2a)$$

$$\begin{aligned} \dot{x}_2 &= -M(x_1)^{-1} \left\{ \frac{1}{2}\dot{M}(x_1)x_2 + S(x_1, x_2)x_2 + g(x_1) \right\} + M(x_1)^{-1}\tau \\ &\triangleq f_2(x_1, x_2) + G_2(x_1)\tau \end{aligned} \quad (2b)$$

$$\mathbf{y} = \mathbf{x}_2 \quad (3)$$

Now taking a storage function equal to the kinetic energy + the potential energy as

$$W(\mathbf{x}) = \frac{1}{2} \mathbf{x}_2^T M(\mathbf{x}_1) \mathbf{x}_2 + U(\mathbf{x}_1) - U(\mathbf{x}_1^*) \quad (4)$$

we calculate its time derivative with use of skew-symmetry of  $S(\mathbf{x}_1, \mathbf{x}_2)$  to obtain

$$\begin{aligned} \dot{W}(\mathbf{x}) &= \frac{\partial}{\partial \mathbf{x}_1} \left\{ \frac{1}{2} \mathbf{x}_2^T M(\mathbf{x}_1) \mathbf{x}_2 \right\} \dot{\mathbf{x}}_1 + \frac{\partial}{\partial \mathbf{x}_2} \left\{ \frac{1}{2} \mathbf{x}_2^T M(\mathbf{x}_1) \mathbf{x}_2 \right\} \dot{\mathbf{x}}_2 + \frac{\partial U(\mathbf{x}_1)}{\partial \mathbf{x}_1} \dot{\mathbf{x}}_1 \\ &= \frac{1}{2} \mathbf{x}_2^T \dot{M}(\mathbf{x}_1) \mathbf{x}_2 + \mathbf{x}_2^T \left\{ -\frac{1}{2} \dot{M}(\mathbf{x}_1) \mathbf{x}_2 - S(\mathbf{x}_1, \mathbf{x}_2) \mathbf{x}_2 - \mathbf{g}(\mathbf{x}_1) + \boldsymbol{\tau} \right\} + \mathbf{g}(\mathbf{x}_1)^T \mathbf{x}_2 \\ &\leq \mathbf{y}^T \boldsymbol{\tau} \end{aligned} \quad (5)$$

Therefore, the robot manipulator is passive with respect to the input  $\boldsymbol{\tau}$  and the output  $\mathbf{y} = \mathbf{x}_2$  (Arimoto, 1996). Thus, the so-called K-Y-P property holds :

$$W_{\mathbf{x}_1}(\mathbf{x}) \mathbf{x}_2 + W_{\mathbf{x}_2}(\mathbf{x}) \mathbf{f}_2(\mathbf{x}_1, \mathbf{x}_2) \leq 0 \quad (6a)$$

$$W_{\mathbf{x}_2}(\mathbf{x}) \mathbf{G}_2(\mathbf{x}_1) = \mathbf{y}^T \quad (6b)$$

Here let us consider a set-point servo problem (a set-point tracking control) with the desired value  $(\mathbf{x}_1, \mathbf{x}_2) = (\mathbf{x}_1^*, \mathbf{0})$ . For that we consider the following system which consists of the robot manipulator (2),(3) and a SPR (strict positive real) element (8).

$$\dot{\mathbf{x}}_1 = \mathbf{x}_2 \quad (7a)$$

$$\begin{aligned} \dot{\mathbf{x}}_2 &= -M(\mathbf{x}_1)^{-1} \left\{ \frac{1}{2} \dot{M}(\mathbf{x}_1) \mathbf{x}_2 + S(\mathbf{x}_1, \mathbf{x}_2) \mathbf{x}_2 + \mathbf{g}(\mathbf{x}_1) \right\} + M(\mathbf{x}_1)^{-1} \boldsymbol{\tau} \\ &\triangleq \mathbf{f}_2(\mathbf{x}_1, \mathbf{x}_2) + \mathbf{G}_2(\mathbf{x}_1) \boldsymbol{\tau} \end{aligned} \quad (7b)$$

$$\dot{\boldsymbol{\xi}} = D \boldsymbol{\xi} + (\mathbf{x}_1^* - \mathbf{x}_1) - \mathbf{x}_2, \quad \boldsymbol{\xi}(0) = \mathbf{0}, \quad D < 0 \quad (8)$$

$$\mathbf{y} = \mathbf{x}_2 \quad (9)$$

And set up a feedback compensator (P·SPR·D controller) :

$$\boldsymbol{\tau} = K_P(\mathbf{x}_1^* - \mathbf{x}_1) + K_S \boldsymbol{\xi} - K_D \mathbf{x}_2 + \mathbf{g}(\mathbf{x}_1^*) \quad (10)$$

where  $K_P, K_S, K_D \in R^{n \times n}$  are all gain matrices being positive definite and diagonal. Here  $\mathbf{g}(\mathbf{x}_1^*)$ , gravity force compensation at the desired value  $\mathbf{x}_1^*$ , corresponds to the so-called manual reset quantity of PID control.

We have the following theorem.

**[Theorem 1]** The closed-loop system (7)~(10) of the robot manipulator with the P·SPR·D control is asymptotically stable at the equilibrium  $(\mathbf{x}_{1e}, \mathbf{x}_{2e}, \boldsymbol{\xi}_e) = (\mathbf{x}_1^*, \mathbf{0}, \mathbf{0})$ , provided that positive definite diagonal matrices  $K_P, K_S, K_D \in R^{n \times n}$  and negative definite diagonal  $D \in R^{n \times n}$  are appropriately chosen.

**(Proof)** At the equilibrium of system (7),(8),(10) hold the following relations.

$$\begin{aligned} 0 &= x_{2e} \\ 0 &= -g(x_{1e}) + \tau_e \\ 0 &= D\zeta_e + (x_1^* - x_{1e}) \end{aligned}$$

Thus it follows that  $(x_{1e}, x_{2e}, \zeta_e) = (x_1^*, 0, 0)$  is an equilibrium point, provided that  $\tau_e = g(x_1^*)$ . Now let us consider a Lyapunov function candidate

$$V(x, \zeta) = W(x) + g(x_1^*)^T (x_1^* - x_1) + \frac{1}{2} \begin{bmatrix} (x_1^* - x_1) \\ \zeta \end{bmatrix}^T \begin{bmatrix} K_P - \bar{K} & \bar{K} \\ \bar{K}^T & K_S - \bar{K} \end{bmatrix} \begin{bmatrix} (x_1^* - x_1) \\ \zeta \end{bmatrix} \quad (11)$$

where  $K_P - \bar{K} > 0$ ,  $K_S - \bar{K} > 0$  and  $\begin{bmatrix} K_P - \bar{K} & \bar{K} \\ \bar{K}^T & K_S - \bar{K} \end{bmatrix}$  is a positive definite matrix. The first term in the right-hand side of (11) is a semi-positive definite function. Since the second term plus the third one is a quadratic function of  $\begin{bmatrix} (x_1^* - x_1) \\ \zeta \end{bmatrix}$  whose quadratic term is with the positive definite matrix, it has the minimum. Accordingly,  $V(x, \zeta)$  is a function bounded below.

Next calculate its time derivative along (7)~(10) using the K-Y-P property (6) to get

$$\begin{aligned} \dot{V}(x, \zeta) &= W_{x_1}(x)x_2 + W_{x_2}(x)f_2(x_1, x_2) + W_{x_2}(x)G_2(x_1)\tau - g(x_1^*)^T x_2 \\ &\quad + \begin{bmatrix} (x_1^* - x_1) \\ \zeta \end{bmatrix}^T \begin{bmatrix} K_P - \bar{K} & \bar{K} \\ \bar{K}^T & K_S - \bar{K} \end{bmatrix} \begin{bmatrix} -\dot{x}_1 \\ \dot{\zeta} \end{bmatrix} \\ &\leq y^T \tau - g(x_1^*)^T x_2 + \begin{bmatrix} (x_1^* - x_1) \\ \zeta \end{bmatrix}^T \begin{bmatrix} K_P - \bar{K} & \bar{K} \\ \bar{K}^T & K_S - \bar{K} \end{bmatrix} \begin{bmatrix} -x_2 \\ D\zeta + (x_1^* - x_1) - x_2 \end{bmatrix} \\ &= x_2^T (K_P(x_1^* - x_1) + K_S\zeta - K_D x_2 + g(x_1^*)) - g(x_1^*)^T x_2 \\ &\quad + \begin{bmatrix} (x_1^* - x_1) \\ \zeta \end{bmatrix}^T \begin{bmatrix} -(K_P - \bar{K})x_2 + \bar{K}D\zeta + \bar{K}(x_1^* - x_1) - \bar{K}x_2 \\ -\bar{K}^T x_2 + (K_S - \bar{K})D\zeta + (K_S - \bar{K})(x_1^* - x_1) - (K_S - \bar{K})x_2 \end{bmatrix} \\ &= x_2^T (K_P(x_1^* - x_1) + K_S\zeta - K_D x_2) + \begin{bmatrix} (x_1^* - x_1) \\ \zeta \end{bmatrix}^T \begin{bmatrix} \bar{K} & \bar{K}D \\ (K_S - \bar{K}) & (K_S - \bar{K})D \end{bmatrix} \begin{bmatrix} (x_1^* - x_1) \\ \zeta \end{bmatrix} \\ &\quad - (x_1^* - x_1)^T \bar{K} x_2 - \zeta^T K_S x_2 \\ &= -x_2^T K_D x_2 + \begin{bmatrix} (x_1^* - x_1) \\ \zeta \end{bmatrix}^T \begin{bmatrix} \bar{K} & \bar{K}D \\ (K_S - \bar{K}) & (K_S - \bar{K})D \end{bmatrix} \begin{bmatrix} (x_1^* - x_1) \\ \zeta \end{bmatrix} \quad (12) \end{aligned}$$

Here we try to make

$$\begin{bmatrix} \bar{K} & \bar{K}D \\ (K_S - \bar{K}) & (K_S - \bar{K})D \end{bmatrix}$$

be negative definite. For that purpose, set  $\bar{K} < 0$ ,  $K_S - \bar{K} = (\bar{K}D)^T$  and  $D < -I$  such that we have  $K_S = (I + D)\bar{K} > 0$ . Then the above matrix becomes

$$\begin{bmatrix} \bar{K} & \bar{K}D \\ (\bar{K}D)^T & \bar{K}D^2 \end{bmatrix}$$

Since the (1,1) element and the (2,2) element are  $\bar{K} < 0$ ,  $\bar{K}D^2 < 0$ , respectively, we can choose  $\bar{K} < 0$  and diagonal  $D < 0$  such that the above matrix becomes negative definite. This can be concluded from the Schur complement also.

Consequently,  $\dot{V}(x, \xi)$  becomes semi-negative definite, and it follows that the P·SPR·D control is stable in the sense of Lyapunov, but it is unknown if asymptotically stable. So we apply LaSalle's invariance principle (LaSalle & Lefschetz, 1961) as below.

Let  $\Omega_c = \{(x, \xi) \mid V(x, \xi) \leq c\}$  and suppose that  $\Omega_c$  is bounded and  $\dot{V}(x, \xi) \leq 0$  in  $\Omega_c$  ( $c$  is a positive number such that  $\dot{V}(x, \xi) \leq 0$ ). Here define  $\Omega_E$  as a set of all points of  $\Omega_c$  satisfying  $\dot{V}(x, \xi) = 0$  and put

$$\Omega_E = \{(x, \xi) \mid \dot{V}(x, \xi) = 0, (x, \xi) \in \Omega_c\}$$

From (12)  $(x, \xi)$  satisfying  $\dot{V}(x, \xi) = 0$  is given as  $x_2 = 0$ ,  $x_1^* - x_1 = 0$ ,  $\xi = 0$ . So we have

$$\Omega_E = \{(x, \xi) \mid x_1 = x_1^*, x_2 = 0, \xi = 0, (x, \xi) \in \Omega_c\}$$

Accordingly, we know from (7),(8),(10) that  $(x, \xi)$  in  $\Omega_E$  consists of only the equilibrium point  $(x_{1e}, x_{2e}, \xi_e) = (x_1^*, 0, 0)$  with  $\tau_e = g(x_1^*)$ . Thus the largest invariance set  $\Omega_M$  in  $\Omega_E$  consists of the equilibrium point  $(x_{1e}, x_{2e}, \xi_e) = (x_1^*, 0, 0)$ . Therefore, by LaSalle's invariance principle all trajectories in  $\Omega_c$  converges to  $\Omega_M$  as  $t \rightarrow \infty$ . Thus  $(x_1^*, 0, 0)$  is asymptotically stable. Namely, it is achieved that  $x_1(t) \rightarrow x_1^*$ ,  $x_2(t) \rightarrow 0$ ,  $\xi(t) \rightarrow 0$ , as  $t \rightarrow \infty$ .  $\square$

**[Remark 1]** It is well-known (Khalil, 2002) that if affine nonlinear system is passive and zero state detectable, then the output feedback control  $u = -Ky$ ,  $K > 0$  asymptotically stabilizes an equilibrium point  $x_e = 0$ . Since the robot manipulator is not zero state detectable, however, one cannot apply this well-known fact to asymptotical stabilization to the origin. In order to stabilize the origin  $(x_1, x_2) = (0, 0)$ , one must apply Theorem 1 letting  $x_1^* = 0$ .

**[Remark 2]** P·SPR·D control of affine nonlinear systems is investigated in (Shimizu, 2008a; 2009b). Its asymptotical stability is proved under the assumption of passivity and zero state detectability.

**[Remark 3]** Although we consider only a rigid robot manipulator in this chapter, elastic joint robot arm is studied in (Shimizu, 2009b; Spong et al., 1992).

Local asymptotical stability of PID control for the robot manipulator was first proved by (Arimoto, 1996; Arimoto & Miyazaki, 1984). For comparison with the P·SPR·D control, its proof based on the K-Y-P property is given in Appendix.

It is well-known (Arimoto, 1996) that PD control + gravity force compensation yields superior control performance. However, in case where the gravity force compensation  $g(x_1^*)$  at the desired value  $x_1^*$  is not available, we can consider the following P·SPR·D+I control instead of (10).

$$\tau = K_P(x_1^* - x_1) + K_S\xi - K_Dx_2 + K_I \int_0^t (x_1^* - x_1(\tau))d\tau \quad (13)$$

i.e.

$$\dot{\xi} = D\xi + (x_1^* - x_1) - x_2, \quad \xi(0) = 0, \quad D < 0$$

$$\dot{z} = x_1^* - x_1, \quad z(0) = 0$$

$$u = K_P(x_1^* - x_1) + K_S\xi - K_D\dot{x}_2 + K_I z$$

Since the stability of transient state is sufficiently guaranteed by the P·SPR·D control, we devise here only a counterplan to remove a steady state error (an off-set). Of course, the P·SPR·D+I control is inferior to the P·SPR·D control with the gravity force compensation. Yet sufficiently satisfactory control performance can be obtained.

### 3. P·SPR·D Control of Redundant Manipulators

Robot manipulators with multi-freedom, the so-called redundant manipulators, can perform complex and flexible operation utilizing the redundancy. Stability analysis of robot manipulators should be made basically in the joint-space coordinates. But actual robot manipulators aim to control direct motion in the task-space. Therefore, it is more convenient for the robot task to represent a model in the task-space showing a manipulator end-point position rather than a model in the joint-space. If joint angles  $q^*$  corresponding to the desired target position  $z^0$  in the task-space can be accurately calculated from inverse kinematics, one may consider stabilization only in the joint-space.

For the redundant manipulators, however, the joint angles  $q^*$  corresponding to the target position  $z^0$  can not be determined uniquely and in addition calculation of inverse kinematics is usually complex and inaccurate. Thus, from a view-point of practice a stable control scheme based on the task-space plus joint-space coordinates is very desirable.

Now let us consider a multi-joint redundant manipulator with  $n$  links. Let  $z \in R^p$  ( $p < n$ ) be the end-point position vector in the task-space. Then one has a relation from the kinematics as (Arimoto, 1996; Khatib, 1987; Spong et al., 1992)

$$z = f(q) = f(x_1) \quad (14)$$

$$\dot{z} = \frac{\partial f(x_1)}{\partial x_1} \dot{x}_1 \triangleq J(x_1) \dot{x}_1 \quad (15)$$

It is easy to calculate forward kinematics  $q \mapsto z$ , but hard to calculate inverse one  $z \mapsto q$ . Namely, given the desired position coordinates  $z^0$ , it is very difficult to determine the joint coordinates  $x_1^* = q^*$  realizing  $z^0$ , as the degree of freedom of redundancy is large.

In order to achieve accurate end-point position control, it is desired to obtain a control method for realizing  $z^0$ , even when the inverse transformation  $x_1^* = f^{-1}(z^0)$  may not be attained correctly. For that purpose we propose a stabilizing control method for the end-point position setting, combining the P·SPR·D control in the joint-space and the P·SPR one in the task-space. Namely, we add the following P·SPR control in the task-space to the P·SPR·D one in the joint-space.

$$\dot{z} = J(x_1) \dot{x}_1 \quad (16)$$

$$\dot{\eta} = D' \eta + (z^0 - z) - J(x_1) \dot{x}_1, \quad \eta(0) = 0, \quad D' < 0 \quad (17)$$

$$\tau' = J(x_1)^T K_P'(z^0 - z) + J(x_1)^T K_S' \eta \quad (18)$$

where  $K_P', K_S' \in R^{p \times p}$  are positive definite diagonal gain matrices. Therefore, actual control input becomes in consideration of task-space coordinates as follows.

$$\begin{aligned} \tau &= K_P(x_1^* - x_1) + K_S \xi - K_D \dot{x}_1 + g(x_1^*) + \tau' \\ &= K_P(x_1^* - x_1) + K_S \xi - K_D \dot{x}_1 + g(x_1^*) + J(x_1)^T K_P'(z^0 - z) + J(x_1)^T K_S' \eta \end{aligned} \quad (19)$$



where the set-point  $x_1^*$  denotes the joint angles corresponding to the desired position  $z^0$ , which is determined from the inverse kinematics. It is not unique, however. Then the following theorem holds.

**[Theorem 2]** The closed-loop system (7)~(9),(16),(17),(19) of the redundant manipulator with P·SPR·D control is asymptotically stable at the equilibrium  $(x_{1e}, x_{2e}, \xi_e, z_e, \eta_e) = (x_1^*, 0, 0, z^0, 0)$ , provided that positive definite diagonal matrices  $K_P, K_S, K_D \in R^{n \times n}$ ,  $K'_P, K'_S \in R^{p \times p}$  and negative definite diagonal matrices  $D \in R^{n \times n}$ ,  $D' \in R^{p \times p}$  are appropriately chosen.

**(Proof)** For simplicity of description, we will state only a part to be added to the proof of Theorem 1.

Consider a Lyapunov function candidate for the overall system

$$V_{total}(x, \xi, z, \eta) = V(x, \xi) + V'(z, \eta) \quad (20)$$

where  $V(x, \xi)$  is given by (11) and  $V'(z, \eta)$  denotes a Lyapunov function candidate corresponding to the additional part (16) ~ (18);

$$V'(z, \eta) = \frac{1}{2} \begin{bmatrix} (z^0 - z) \\ \eta \end{bmatrix}^T \begin{bmatrix} K'_P - \bar{K}' & \bar{K}' \\ \bar{K}'^T & K'_S - \bar{K}' \end{bmatrix} \begin{bmatrix} (z^0 - z) \\ \eta \end{bmatrix} \quad (21)$$

where  $K'_P - \bar{K}' > 0$ ,  $K'_S - \bar{K}' > 0$  and  $\begin{bmatrix} K'_P - \bar{K}' & \bar{K}' \\ \bar{K}'^T & K'_S - \bar{K}' \end{bmatrix}$  is a positive definite matrix.

Next calculate a time derivative of (20) along (7)~(9),(16),(17),(19) to get

$$\dot{V}_{total}(x, \xi, z, \eta) = \dot{V}(x, \xi) + \dot{V}'(z, \eta) \quad (22)$$

But  $\dot{V}(x, \xi)$  has been evaluated by (12) except for a part of  $y^T \tau'$ , already, so we calculate only the remained part as follows.

$$\begin{aligned} & y^T \tau' + \dot{V}'(z, \eta) \\ & \leq y^T \tau' + \begin{bmatrix} (z^0 - z) \\ \eta \end{bmatrix}^T \begin{bmatrix} K'_P - \bar{K}' & \bar{K}' \\ \bar{K}'^T & K'_S - \bar{K}' \end{bmatrix} \begin{bmatrix} (\dot{z}^0 - \dot{z}) \\ \dot{\eta} \end{bmatrix} \\ & = x_2^T \tau' + \begin{bmatrix} (z^0 - z) \\ \eta \end{bmatrix}^T \begin{bmatrix} K'_P - \bar{K}' & \bar{K}' \\ \bar{K}'^T & K'_S - \bar{K}' \end{bmatrix} \begin{bmatrix} -J(x_2)x_2 \\ D'\eta + (z^0 - z) - J(x_1)x_2 \end{bmatrix} \\ & = x_2^T (J(x_1)^T K'_P (z^0 - z) + J(x_1)^T K'_S \eta) \\ & \quad + \begin{bmatrix} (z^0 - z) \\ \eta \end{bmatrix}^T \begin{bmatrix} -(K'_P - \bar{K}')J(x_1)x_2 + \bar{K}' D'\eta + \bar{K}'(z^0 - z) - \bar{K}' J(x_1)x_2 \\ -\bar{K}'^T J(x_1)x_2 + (K'_S - \bar{K}')D'\eta + (K'_S - \bar{K}')(z^0 - z) - (K'_S - \bar{K}')J(x_1)x_2 \end{bmatrix} \\ & = x_2^T (J(x_1)^T K'_P (z^0 - z) + J(x_1)^T K'_S \eta) + \begin{bmatrix} (z^0 - z) \\ \eta \end{bmatrix}^T \begin{bmatrix} \bar{K}' & \bar{K}' D' \\ (K'_S - \bar{K}') & (K'_S - \bar{K}')D' \end{bmatrix} \begin{bmatrix} (z^0 - z) \\ \eta \end{bmatrix} \\ & \quad - (z^0 - z)^T K'_P J(x_1)x_2 - \eta^T K'_S J(x_1)x_2 \\ & = \begin{bmatrix} (z^0 - z) \\ \eta \end{bmatrix}^T \begin{bmatrix} \bar{K}' & \bar{K}' D' \\ (K'_S - \bar{K}') & (K'_S - \bar{K}')D' \end{bmatrix} \begin{bmatrix} (z^0 - z) \\ \eta \end{bmatrix} \quad (23) \end{aligned}$$



Therefore, combining (12) and (23), we have

$$\begin{aligned} \dot{V}_{total}(\mathbf{x}, \boldsymbol{\xi}, \mathbf{z}, \boldsymbol{\eta}) \leq & -\mathbf{x}_2^T K_D \mathbf{x}_2 + \begin{bmatrix} (\mathbf{x}_1^* - \mathbf{x}_1) \\ \boldsymbol{\xi} \end{bmatrix}^T \begin{bmatrix} \bar{K} & \bar{K}D \\ (K_S - \bar{K}) & (K_S - \bar{K})D \end{bmatrix} \begin{bmatrix} (\mathbf{x}_1^* - \mathbf{x}_1) \\ \boldsymbol{\xi} \end{bmatrix} \\ & + \begin{bmatrix} (\mathbf{z}^o - \mathbf{z}) \\ \boldsymbol{\eta} \end{bmatrix}^T \begin{bmatrix} \bar{K}' & \bar{K}'D' \\ (K'_S - \bar{K}') & (K'_S - \bar{K}')D' \end{bmatrix} \begin{bmatrix} (\mathbf{z}^o - \mathbf{z}) \\ \boldsymbol{\eta} \end{bmatrix} \end{aligned} \quad (24)$$

The third term in the right-hand side can be made negative definite by the similar argument in Theorem 1. Hence the function (24) is semi-negative definite.

By the way, when  $\boldsymbol{\tau} = \boldsymbol{\tau}_e = \mathbf{g}(\mathbf{x}_1^*)$ , it is obvious that an equilibrium of (7),(8),(16),(17),(19) becomes  $(\mathbf{x}_{1e}, \mathbf{x}_{2e}, \boldsymbol{\xi}_e, \mathbf{z}_e, \boldsymbol{\eta}_e) = (\mathbf{x}_1^*, \mathbf{0}, \mathbf{0}, \mathbf{z}^o, \mathbf{0})$ . Therefore, by the similar argument in Theorem 1, we can show that the equilibrium  $(\mathbf{x}_1^*, \mathbf{0}, \mathbf{0}, \mathbf{z}^o, \mathbf{0})$  is asymptotically stable by LaSalle's invariance principle. Namely, it is achieved that  $\mathbf{x}_1(t) \rightarrow \mathbf{x}_1^*$ ,  $\mathbf{x}_2(t) \rightarrow \mathbf{0}$ ,  $\boldsymbol{\xi}(t) \rightarrow \mathbf{0}$ ,  $\mathbf{z}(t) \rightarrow \mathbf{z}^o$ ,  $\boldsymbol{\eta}(t) \rightarrow \mathbf{0}$ , as  $t \rightarrow \infty$ .  $\square$

**AyCollorary 1A**z Theorem 2 holds also with setting  $K'_S = 0$  and  $\boldsymbol{\tau}' = J(\mathbf{x}_1)^T K'_P(\mathbf{z}^o - \mathbf{z})$ .

Meanwhile, when  $n > p$  in the redundant manipulator, we can set some joint angles  $q_i, i \in I$  at arbitrary values  $x_{1i}^* = q_i^*, i \in I$  within the possible freedom (the number of elements of  $I$  is less than  $n - p$ ). In this case define a vector  $\hat{\mathbf{x}}_1 \in R^n$  as  $\{\hat{x}_{1i} = x_{1i}, i \in I, \hat{x}_{1i} = 0, i \notin I\}$  and  $\{\hat{x}_{1i}^* = x_{1i}^*, i \in I, \hat{x}_{1i}^* = 0, i \notin I\}$  and let us modify the control law (16),(17),(19) as follows.

$$\dot{\boldsymbol{\xi}} = D\boldsymbol{\xi} + (\hat{\mathbf{x}}_1^* - \hat{\mathbf{x}}_1) - \hat{\mathbf{x}}_2, \quad \boldsymbol{\xi}(0) = \mathbf{0}, \quad D < 0 \quad (25)$$

$$\dot{\mathbf{z}} = J(\mathbf{x}_1)\mathbf{x}_2 \quad (26)$$

$$\dot{\boldsymbol{\eta}} = D'\boldsymbol{\eta} + (\mathbf{z}^o - \mathbf{z}) - J(\mathbf{x}_1)\mathbf{x}_2, \quad \boldsymbol{\eta}(0) = \mathbf{0}, \quad D' < 0 \quad (27)$$

$$\boldsymbol{\tau} = K_P(\hat{\mathbf{x}}_1^* - \hat{\mathbf{x}}_1) + K_S\boldsymbol{\xi} - K_D\hat{\mathbf{x}}_2 + \mathbf{g}(\mathbf{x}_1^*) + J(\mathbf{x}_1)^T K'_P(\mathbf{z}^o - \mathbf{z}) + J(\mathbf{x}_1)^T K'_S\boldsymbol{\eta} \quad (28)$$

It is then noted that elements of  $K_P$  and  $K_S$  corresponding to  $x_{1i}, i \notin I$  do not give any effect. In this case, asymptotical stability in the subspace of joint coordinates  $x_i, i \in I$  is guaranteed such that  $x_{1i}(t) \rightarrow x_{1i}^*, i \in I$  as  $t \rightarrow \infty$ . But joint angles  $x_{1i}, i \notin I$  are not known where to converge, although stable in the sense of Lyapunov.

Theorem 2 does not take damping (Derivative action) in the task-space in consideration. However, damping  $-K_D\mathbf{x}_2$  in the joint-space contributes to it indirectly.

In order to add the damping in the task-space to control input  $\boldsymbol{\tau}'$ , we can add Derivative action term  $-XK'_D\dot{\mathbf{z}}$  to  $\boldsymbol{\tau}'$ . Although we can calculate the matrix  $X$  theoretically, however, it is not of practical use because of too much complexity.

On the other hand, under the situation of  $\ddot{\mathbf{q}} = \dot{\mathbf{x}}_2 \approx \mathbf{0}$  one can prove asymptotical stability, even if  $-J(\mathbf{x}_1)^T K'_D\dot{\mathbf{z}}$  is added to (18). But we do not know whether the damping is effective or not, as  $\dot{\mathbf{x}}_2 \approx \mathbf{0}$  is not assumed.

#### 4. $L_2$ -Gain Disturbance Attenuation Problem

In this section we study  $L_2$ -gain disturbance attenuation problem under the existence of disturbance  $w$ . Let us consider again the following cascaded system of the robot manipulator

and the SPR element.

$$\dot{x}_1 = x_2 \quad (29a)$$

$$\begin{aligned} \dot{x}_2 &= -M(x_1)^{-1} \left\{ \frac{1}{2} \dot{M}(x_1)x_2 + S(x_1, x_2)x_2 + g(x_1) \right\} + M(x_1)^{-1}\tau + L(x)w \\ &\triangleq f_2(x_1, x_2) + G_2(x_1)\tau + L(x)w \end{aligned} \quad (29b)$$

$$\dot{\xi} = D\xi + (x_1^* - x_1) - x_2, \quad D < 0 \quad (30)$$

$$y = x_2 \quad (31)$$

where  $w \in R^l$  is the disturbance vector.

And set up a feedback compensator (P·SPR·D controller) :

$$\tau = K_P(x_1^* - x_1) + K_S\xi - K_Dx_2 + g(x_1^*) \quad (32)$$

where  $K_P, K_S, K_D \in R^{n \times n}$  are all positive definite diagonal matrices. Here  $g(x_1^*)$ , gravity force compensation at the desired value  $x_1^*$ , corresponds to the manual reset quantity  $m_0$ .

The  $L_2$ -gain disturbance attenuation problem is defined to obtain the P·SPR·D control such that the closed-loop system satisfies the following two conditions under the given disturbance attenuation level  $\gamma > 0$ .

**P1.** When  $w = 0$ , the closed-loop system is asymptotically stable at the equilibrium  $(x_{1e}, x_{2e}, \xi_e) = (x_1^*, 0, 0)$ .

**p2.** When  $x(0) = 0$ , the following inequality holds for arbitrarily given  $T > 0$ .

$$\int_0^T \|y(t)\|^2 dt \leq \gamma^2 \int_0^T \|w(t)\|^2 dt$$

It is noticed that P2 is equivalent to having  $L_2$  gain below  $\gamma$  when  $x(0) = 0$ , that is,  $\|y\|_2 \leq \gamma^2 \|w\|_2$ . It implies that for all  $w \in L_2[0, T]$  and for the supply rate  $s(y, w) = \frac{1}{2} \{ \gamma^2 w^T w - y^T y \}$ , the following  $\gamma$ -dissipation inequality holds (van der Schaft, 2000).

$$\dot{V}(x, \xi) \leq \frac{1}{2} \{ \gamma^2 w^T w - y^T y \} \quad (33)$$

The following theorem solves the  $L_2$ -gain disturbance attenuation problem.

**[Theorem 3]** Suppose that  $W(x)$  and  $L(x)$  satisfy the matching condition

$$W_x(x)L(x) = y^T S(x)^T \quad (34)$$

where  $S(x) \in R^{l \times n}$  denotes the function matrix and  $S(x)^T S(x) = I_n$ . Then the closed-loop system (29)~(32) satisfies P2, provided that positive definite diagonal matrices  $K_P, K_S, K_D \in R^{n \times n}$  and negative definite diagonal matrix  $D \in R^{n \times n}$  are appropriately chosen and  $K_D \geq \frac{1}{2}(1 + \frac{1}{\gamma^2})I_n$ . Namely, it possesses  $L_2$  gain less than  $\gamma$  (i.e.,  $\gamma$ -dissipation inequality holds.)

Furthermore, by the P·SPR·D control (32) the closed-loop system satisfies P1 so that  $(x_{1e}, x_{2e}, \xi_e) = (x_1^*, 0, 0)$  is asymptotically stable.

**(Proof)** To prove that the  $\gamma$ -dissipation inequality holds, make the following calculation for a storage function (11) (semi-positive definite function).

$$\begin{aligned} & \dot{V}(x, \xi) + \frac{1}{2}\{\mathbf{y}^T \mathbf{y} - \gamma^2 \mathbf{w}^T \mathbf{w}\} \\ &= W_{x_1}(x)x_2 + W_{x_2}(x)f_2(x_1, x_2) + W_{x_2}(x)G_2(x_1)\boldsymbol{\tau} + W_{x_2}(x)L(x)\mathbf{w} - \mathbf{g}(x_1^*)^T x_2 \\ & \quad + \begin{bmatrix} (x_1^* - x_1) \\ \xi \end{bmatrix}^T \begin{bmatrix} K_P - \bar{K} & \bar{K} \\ \bar{K}^T & K_S - \bar{K} \end{bmatrix} \begin{bmatrix} (\dot{x}_1^* - \dot{x}_1) \\ \dot{\xi} \end{bmatrix} + \frac{1}{2}\{\mathbf{y}^T \mathbf{y} - \gamma^2 \mathbf{w}^T \mathbf{w}\} \end{aligned}$$

Here using the K-Y-P property (6) and the control (32) and the matching condition (34),

we have

$$\begin{aligned} & \dot{V}(x, \xi) + \frac{1}{2}\{\mathbf{y}^T \mathbf{y} - \gamma^2 \mathbf{w}^T \mathbf{w}\} \\ & \leq \mathbf{y}^T \boldsymbol{\tau} + \mathbf{y}^T S(x)^T \mathbf{w} - \mathbf{g}(x_1^*)^T x_2 + \begin{bmatrix} (x_1^* - x_1) \\ \xi \end{bmatrix}^T \begin{bmatrix} K_P - \bar{K} & \bar{K} \\ \bar{K}^T & K_S - \bar{K} \end{bmatrix} \begin{bmatrix} D\xi + (x_1^* - x_1) - x_2 \\ D\xi + (x_1^* - x_1) - x_2 \end{bmatrix} \\ & \quad + \frac{1}{2}\{\mathbf{y}^T \mathbf{y} - \gamma^2 \mathbf{w}^T \mathbf{w}\} \\ &= x_2^T (K_P(x_1^* - x_1) + K_S \xi - K_D x_2 + \mathbf{g}(x_1^*)) + x_2^T S(x)^T \mathbf{w} - \mathbf{g}(x_1^*)^T x_2 \\ & \quad + \begin{bmatrix} (x_1^* - x_1) \\ \xi \end{bmatrix}^T \begin{bmatrix} -(K_P - \bar{K})x_2 + \bar{K}D\xi + \bar{K}(x_1^* - x_1) - \bar{K}x_2 \\ -\bar{K}^T x_2 + (K_S - \bar{K})D\xi + (K_S - \bar{K})(x_1^* - x_1) - (K_S - \bar{K})x_2 \end{bmatrix} \\ & \quad + \frac{1}{2}\{\mathbf{y}^T \mathbf{y} - \gamma^2 \mathbf{w}^T \mathbf{w}\} \\ &= x_2^T (K_P(x_1^* - x_1) + K_S \xi - K_D x_2) + \mathbf{y}^T S(x)^T \mathbf{w} \\ & \quad + \begin{bmatrix} (x_1^* - x_1) \\ \xi \end{bmatrix}^T \begin{bmatrix} \bar{K} & \bar{K}D \\ (K_S - \bar{K}) & (K_S - \bar{K})D \end{bmatrix} \begin{bmatrix} (x_1^* - x_1) \\ \xi \end{bmatrix} - (x_1^* - x_1)^T K_P x_2 - \xi^T K_S x_2 \\ & \quad + \frac{1}{2}\{\mathbf{y}^T \mathbf{y} - \gamma^2 \mathbf{w}^T \mathbf{w}\} \\ &= -x_2^T K_D x_2 + \mathbf{y}^T S(x)^T \mathbf{w} + \begin{bmatrix} (x_1^* - x_1) \\ \xi \end{bmatrix}^T \begin{bmatrix} \bar{K} & \bar{K}D \\ (K_S - \bar{K}) & (K_S - \bar{K})D \end{bmatrix} \begin{bmatrix} (x_1^* - x_1) \\ \xi \end{bmatrix} \\ & \quad - \frac{1}{2} \left\{ \frac{1}{\gamma} \mathbf{y}^T S(x)^T - \gamma \mathbf{w}^T \right\} \left\{ \frac{1}{\gamma} S(x) \mathbf{y} - \gamma \mathbf{w} \right\} + \frac{1}{2} \mathbf{y}^T \mathbf{y} + \frac{1}{2} \frac{1}{\gamma^2} \mathbf{y}^T S(x)^T S(x) \mathbf{y} \\ & \quad - \frac{1}{2} \mathbf{w}^T S(x) \mathbf{y} - \frac{1}{2} \mathbf{y}^T S(x)^T \mathbf{w} \\ &= \begin{bmatrix} (x_1^* - x_1) \\ \xi \end{bmatrix}^T \begin{bmatrix} \bar{K} & \bar{K}D \\ (K_S - \bar{K}) & (K_S - \bar{K})D \end{bmatrix} \begin{bmatrix} (x_1^* - x_1) \\ \xi \end{bmatrix} - \mathbf{y}^T \left\{ K_D - \frac{1}{2} \left( 1 + \frac{1}{\gamma^2} \right) I_n \right\} \mathbf{y} \\ & \quad - \frac{1}{2} \left\{ \frac{1}{\gamma} S(x) \mathbf{y} - \gamma \mathbf{w} \right\}^T \left\{ \frac{1}{\gamma} S(x) \mathbf{y} - \gamma \mathbf{w} \right\} \end{aligned}$$

The first term in the right-hand side is negative definite, as mentioned below (12). Hence, using  $K_D \geq \frac{1}{2}(1 + \frac{1}{\gamma^2})I_n$ ,

$$\dot{V}(x,\xi) + \frac{1}{2}\{y^Ty - \gamma^2w^Tw\} \leq -\frac{1}{2}\left\{\frac{1}{\gamma}S(x)y - \gamma w\right\}^T \left\{\frac{1}{\gamma}S(x)y - \gamma w\right\} \leq 0 \tag{35}$$

Consequently,  $\gamma$ -dissipation inequality (33) holds, and so it follows that we have  $L_2$  gain below  $\gamma$ .  
When  $w = 0$ , P1 has been already concluded by Theorem 2. □

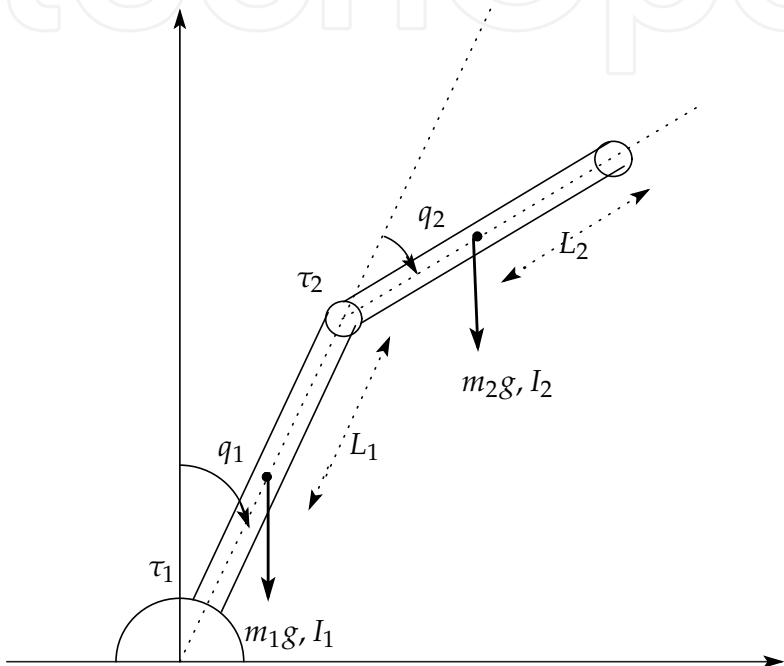


Fig. 1. 2-Link Manipulator

5. Simulation

Let us apply the P·SPR·D control of robot manipulator developed in Section 2 to a two-link manipulator depicted in Fig.1. Here generalized coordinates  $q_1, q_2$  are relative joint angles, and  $x_{11} \triangleq q_1$  denotes the perpendicular angle (angle from vertical line) of link 1 and  $x_{12} \triangleq q_2$  relative angle of link 2 from link 1,  $\tau_1$  and  $\tau_2$  denote the torque of each joint acting clockwise.  $L_1, L_2, m_1, m_2, I_1, I_2$  denote the length, the mass and the inertia moment of each link, respectively.  
A numerical example of two-link manipulator is given as follows.

$$\begin{bmatrix} \dot{x}_{11} \\ \dot{x}_{12} \\ \dot{x}_{21} \\ \dot{x}_{22} \end{bmatrix} = \begin{bmatrix} x_{21} \\ x_{22} \\ f_{21}(x_1, x_2) + G_{211}(x_1)\tau_1 + G_{212}(x_1)\tau_2 \\ f_{22}(x_1, x_2) + G_{221}(x_1)\tau_1 + G_{222}(x_1)\tau_2 \end{bmatrix}$$

where

$$\begin{aligned} f_{21}(x_1, x_2) &\triangleq \frac{-1}{\det M} [1.05 \{ (-6x_{21}x_{22} - 3x_{22}^2) \sin x_{12} + 5x_{21} - 117.6 \sin x_{11} \\ &\quad - 14.7 \sin(x_{11} + x_{12}) \} - (1 + 3 \cos x_{12})(3x_{21}^2 \sin x_{12} + 5x_{22} - 14.7 \sin(x_{11} + x_{12}))] \\ f_{22}(x_1, x_2) &\triangleq \frac{-1}{\det M} [(-1 - 3 \cos x_{12}) \{ (-6x_{21}x_{22} - 3x_{22}^2) \sin x_{12} + 5x_{21} - 117.6 \sin x_{11} \\ &\quad - 14.7 \sin(x_{11} + x_{12}) \} + (21.2 + 6 \cos x_{12})(3x_{21}^2 \sin x_{12} + 5x_{22} - 14.7 \sin(x_{11} + x_{12}))] \\ G_{211}(x_1) &\triangleq \frac{1.05}{\det M}, \quad G_{212}(x_1) \triangleq \frac{1}{\det M} (1 - 3 \cos x_{12}), \\ G_{221}(x_1) &\triangleq \frac{1}{\det M} (-1 - 3 \cos x_{12}), \quad G_{222}(x_1) \triangleq \frac{1}{\det M} (21.2 + 6 \cos x_{12}) \end{aligned}$$

and  $\det M \triangleq 21.26 + 0.3 \cos x_{12} - 9(\cos x_{12})^2$ .

Further,  $g(x_1)$  is also given as

$$\begin{bmatrix} g_1(x_1) \\ g_2(x_1) \end{bmatrix} = \begin{bmatrix} -117.6 \sin x_{11} - 14.7 \sin(x_{11} + x_{12}) \\ -14.7 \sin(x_{11} + x_{12}) \end{bmatrix}$$

Applying Theorem 1, let us solve a set-point servo problem with the desired value  $x_1^* = (1.5, 1)^T$ . We set the SPR element as (8) and take an initial state as  $(x_1(0), x_2(0)) = (0, 0)$ . The simulation results is shown in Fig.2, when  $D = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$ ,  $K_P = \begin{bmatrix} 180 & 0 \\ 0 & 180 \end{bmatrix}$ ,  $K_S = \begin{bmatrix} 40 & 0 \\ 0 & 40 \end{bmatrix}$ ,  $K_D = \begin{bmatrix} 60 & 0 \\ 0 & 60 \end{bmatrix}$ . We see that the convergence speed is very quick and the overshoot is very little.

Furthermore, as mentioned in Remark 1, the regulation problem (asymptotical stabilization to the origin) can be solved by setting  $x_1^* = 0$ . At this time  $m_0 = g(x_1^*)$  is zero. The simulation results is shown in Fig.3.

Fig.4 shows the simulation results of P·SPR·D+I control in case where gravity force compensation  $g(x_1^*)$  is not available. Here  $K_I$  is given as  $\begin{bmatrix} 80 & 0 \\ 0 & 80 \end{bmatrix}$ . Fig.5 shows the simulation results for the regulation problem.

It is seen that the P·SPR·D control is superior to the P·SPR·D+I control (Notice scales of y-axis). Nevertheless, the control performance by the P·SPR·D+I control is also satisfactory enough. Meanwhile, ordinary PID control(with  $mv_0 = 0$ ) is represented as follows.

$$\tau = K_P(x_1^* - x_1) + K_I \int_0^t (x_1^* - x_1(\tau)) d\tau - K_D \dot{x}_2$$

i.e.

$$\dot{z} = x_1^* - x_1, \quad z(0) = 0$$

$$u = K_P(x_1^* - x_1) + K_I z - K_D \dot{x}_2$$

The simulation results by the PID control with  $K_P = \begin{bmatrix} 180 & 0 \\ 0 & 180 \end{bmatrix}$ ,  $K_I = \begin{bmatrix} 40 & 0 \\ 0 & 40 \end{bmatrix}$ ,  $K_D = \begin{bmatrix} 60 & 0 \\ 0 & 60 \end{bmatrix}$  are shown in Fig.6 and Fig.7 for the set-point servo problem and the regulation

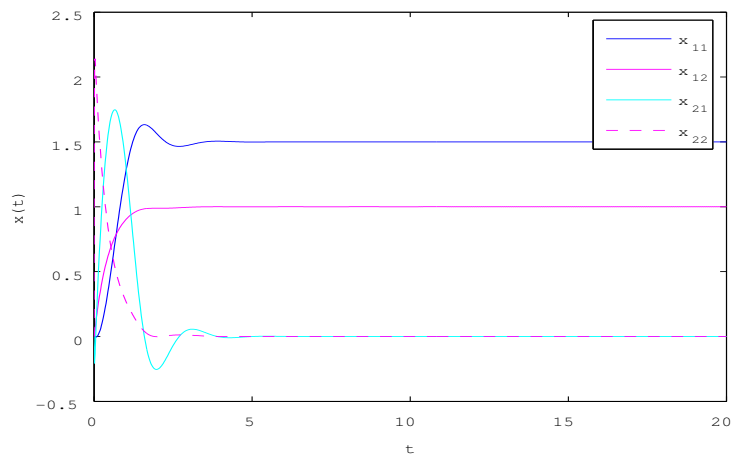


Fig. 2. P-SPR-D Control

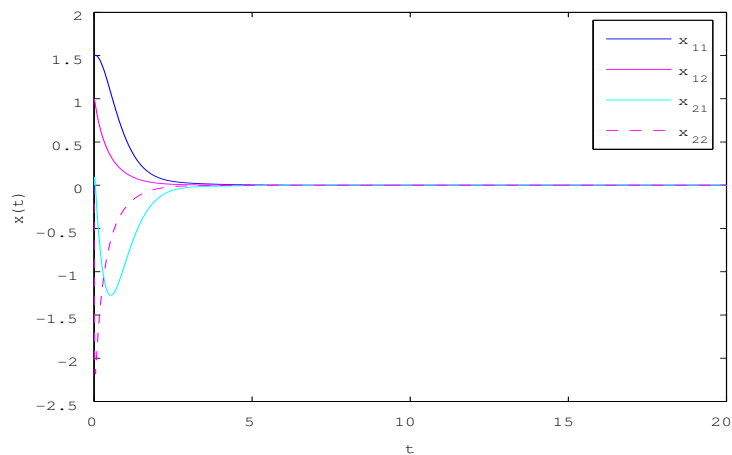


Fig. 3. P-SPR-D Control

one, respectively. It is observed that though the convergence was attained by the ordinary PID control also, the convergence speed is slower than the P-SPR-D+I control. Of course control performance changes depending on controller parameters  $K_P, K_S, K_D, D$ . However, it was known that the proposed methods attained always much better performances than the ordinary PID control. Comparing these three cases, we can say that the P-SPR-D control is the best in regard to response speed, overshoot and steady state error. This indicates that the P-SPR-D or P-SPR-D+I control possesses a possibility of a new and promising control scheme.

Note that nothing has been mentioned in regard to controller parameter adjustment. The values of  $K_P, K_S, K_D$  used in the simulations are the same one with almost optimum values of  $K_P, K_I, K_D$  for the ordinary PID control which were obtained by trial and error. Of course the control performance depends on the parameter values, and so there is a room of argument for further improvement.

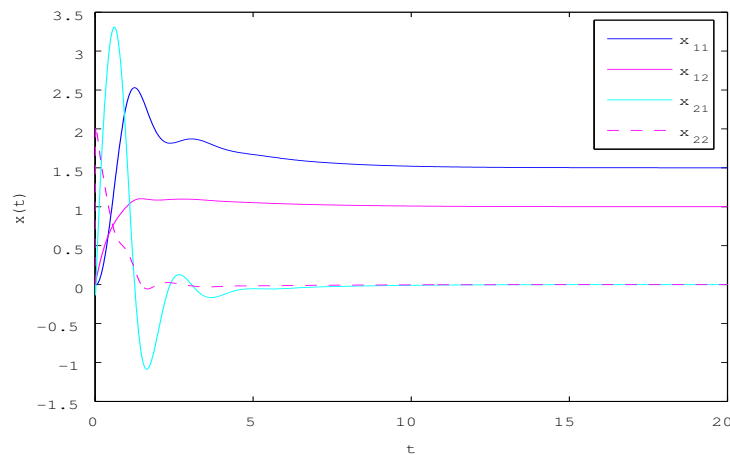


Fig. 4. P·SPR·D+I Control

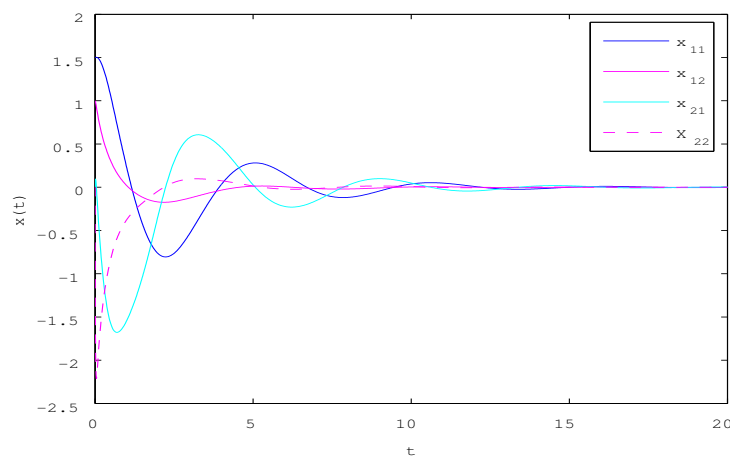


Fig. 5. P·SPR·D+I Control

## 6. Conclusion

Based on the passivity theory and LaSalle's invariance principle, we studied first the set-point servo problem for the robot manipulators by the P·SPR·D and P·SPR·D+I control. Next we investigated a stabilizing control method for the end-point position setting of redundant manipulator, combining the P·SPR·D control in the joint-space and the P·SPR one in the task-space. The effectiveness of the proposed methods are demonstrated with a two-link manipulator. We showed the simulation results of the P·SPR·D+ $g(x_1^*)$  control and P·SPR·D+I control by which very excellent control performances were obtained. Further, the  $L_2$ -gain disturbance attenuation problem was studied also.

The P·SPR·D or P·SPR·D+I control can be said to be a new general control scheme and the use of SPR element as a part of controller possesses an advantage from a passivity-based design point of view. In particular the SPR element contributes powerfully to stabilization of the closed-loop system. They can be applied widely to linear systems and/or affine nonlinear systems also. The optimum adjustment of controller parameters is left as a future topic. Implementation of the P·SPR·D control is not difficult with a digital processor.



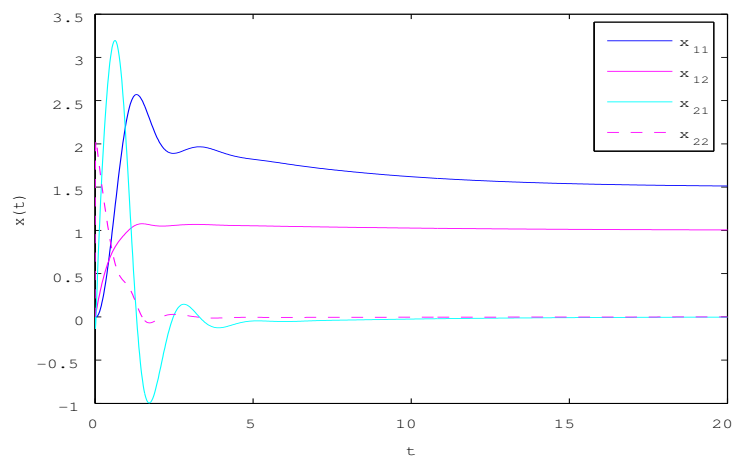


Fig. 6. Ordinary PID Control

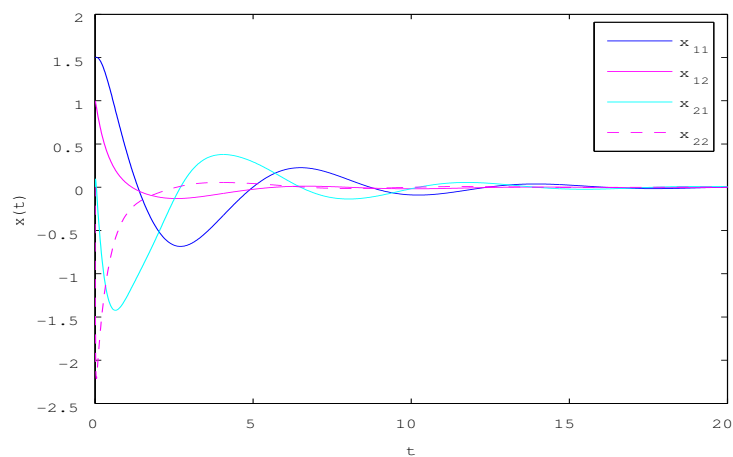


Fig. 7. Ordinary PID Control

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## 8. Appendix Case Where Ordinary PID Control

Let us consider the robot manipulator (2),(3). As is well-known, the robot manipulator is passive with respect to input  $\tau$  and output  $y$ , and hence the K-Y-P property (6) holds. We study here a set-point servo problem with the desired value  $(x_1, x_2) = (x_1^*, 0)$ , and set up an ordinary PID controller

$$\dot{z} = (x_1^* - x_1) \quad (36)$$

$$\tau = K_P(x_1^* - x_1) + K_I z - K_D x_2 \quad (37)$$

where  $K_P, K_I, K_D \in R^{n \times n}$  are positive definite diagonal gain matrices.

Below we prove asymptotical stability of the closed-loop system, applying LaSalle's invariance principle.

An equilibrium of the closed-loop system (2),(36),(37) satisfies

$$\begin{aligned} 0 &= x_{2e} \\ 0 &= -g(x_{1e}) + K_P(x_1^* - x_{1e}) + K_I z_e \\ 0 &= (x_1^* - x_{1e}) \end{aligned} \quad (38)$$

, hence  $(x_{1e}, x_{2e}, z_e) = (x_1^*, 0, \bar{z})$ ,  $\bar{z} = K_I^{-1}g(x_1^*)$  becomes an equilibrium point. Now consider a Lyapunov function candidate

$$\begin{aligned} V(x, z) = & W(x) + g(x_1^*)^T(x_1^* - x_1) + \frac{1}{2}(x_1^* - x_1)^T K_P(x_1^* - x_1) \\ & + (x_1^* - x_1)^T K_I(z - \bar{z}) + \frac{1}{2}\alpha(z - \bar{z})^T K_I(z - \bar{z}) \\ & - \alpha(x_1^* - x_1)^T M(x_1)x_2 \end{aligned} \quad (39)$$

where  $W(x) = \frac{1}{2}x_2^T M(x_1)x_2 + U(x_1) - U(x_1^*)$ ,  $\alpha > 0$ .

It can be proved that  $V(x, z)$  is a function bounded below in the neighborhood of  $(x^*, 0, \bar{z})$ .

Calculate a time derivative of  $V(x, z)$  along (2),(3),(36),(37), using the K-Y-P property (6), to obtain

$$\begin{aligned} \dot{V}(x, z) = & W_{x_1}(x)x_2 + W_{x_2}(x)\{f_2(x_1, x_2) + G_2(x_1)\tau\} - g(x_1^*)^T x_2 \\ & + (x_1^* - x_1)^T K_P(-x_2) - x_2^T K_I(z - \bar{z}) + (x_1^* - x_1)^T K_I \dot{z} + \alpha(z - \bar{z})^T K_I \dot{z} \\ & + \alpha x_2^T M(x_1)x_2 - \alpha(x_1^* - x_1)^T \dot{M}(x_1)x_2 - \alpha(x_1^* - x_1)^T M(x_1)\dot{x}_2 \\ \leq & y^T \tau - g(x_1^*)^T x_2 - (x_1^* - x_1)^T K_P x_2 - x_2^T K_I(z - \bar{z}) \\ & + (x_1^* - x_1)^T K_I(x_1^* - x_1) + \alpha(z - \bar{z})^T K_I(x_1^* - x_1) + \alpha x_2^T M(x_1)x_2 \\ & - \alpha(x_1^* - x_1)^T \dot{M}(x_1)x_2 - \alpha(x_1^* - x_1)^T M(x_1)\{f_2(x_1, x_2) + G_2(x_1)\tau\} \\ = & x_2^T (K_P(x_1^* - x_1) + K_I z - K_D x_2) - g(x_1^*)^T x_2 - (x_1^* - x_1)^T K_P x_2 \\ & - x_2^T K_I(z - \bar{z}) + (x_1^* - x_1)^T K_I(x_1^* - x_1) + \alpha(z - \bar{z})^T K_I(x_1^* - x_1) \\ & + \alpha x_2^T M(x_1)x_2 - \alpha(x_1^* - x_1)^T \dot{M}(x_1)x_2 \\ & - \alpha(x_1^* - x_1)^T \left\{ \frac{1}{2}\dot{M}(x_1)x_2 + S(x_1, x_2)x_2 + g(x_1) \right\} \\ & - \alpha(x_1^* - x_1)^T (K_P(x_1^* - x_1) + K_I z - K_D x_2) \\ = & -x_2^T (K_D - \alpha M(x_1))x_2 - (x_1^* - x_1)^T (\alpha K_P - K_I)(x_1^* - x_1) \\ & - \alpha g(x_1^*)^T (x_1^* - x_1) - \alpha(x_1^* - x_1)^T \dot{M}(x_1)x_2 \\ & + \alpha(x_1^* - x_1)^T \left\{ \frac{1}{2}\dot{M}(x_1)x_2 + S(x_1, x_2)x_2 + g(x_1) \right\} + \alpha(x_1^* - x_1)^T K_D x_2 \\ = & -x_2^T (K_D - \alpha M(x_1))x_2 - (x_1^* - x_1)^T (\alpha K_P - K_I)(x_1^* - x_1) \\ & + \alpha(g(x_1) - g(x_1^*))^T (x_1^* - x_1) + \alpha(x_1^* - x_1)^T Q(x_1, x_2; K_D)x_2 \end{aligned}$$

where  $Q(x_1, x_2; K_D) \triangleq -\frac{1}{2}\dot{M}(x_1) + S(x_1, x_2) + K_D$ .

Here assume for  $\beta > 1$  that

$$(x_1^* - x_1)^T K_P(x_1^* - x_1) \geq \beta(g(x_1) - g(x_1^*))^T (x_1^* - x_1),$$

then we have

$$\begin{aligned}
 \dot{V}(x, z) &\leq -x_2^T (K_D - \alpha M(x_1)) x_2 - (x_1^* - x_1)^T (\alpha K_P - K_I) (x_1^* - x_1) \\
 &\quad + \frac{\alpha}{\beta} (x_1^* - x_1)^T K_P (x_1^* - x_1) + \alpha (x_1^* - x_1)^T Q(x_1, x_2; K_D) x_2 \\
 &= -x_2^T (K_D - \alpha M(x_1)) x_2 \\
 &\quad - \begin{bmatrix} x_1^* - x_1 \\ x_2 \end{bmatrix}^T \begin{bmatrix} (\alpha - \frac{\alpha}{\beta}) K_P - K_I & -\frac{1}{2} \alpha Q(x_1, x_2; K_D) \\ -\frac{1}{2} \alpha Q(x_1, x_2; K_D)^T & K_D - \alpha M(x_1) \end{bmatrix} \begin{bmatrix} x_1^* - x_1 \\ x_2 \end{bmatrix} \\
 &\leq 0
 \end{aligned} \tag{40}$$

By supposing that  $x_2$  exists in the neighborhood of  $x_2 = 0$ , spectral radius of  $Q(x_1, x_2; K_D)$  can be considered within a certain value. When  $x_2$  exists within that bounds, by taking  $\alpha$  sufficiently small and  $K_I > 0$  appropriately small for the given  $\beta$ , we can make the matrix

$$\begin{bmatrix} (\alpha - \frac{\alpha}{\beta}) K_P - K_I & -\frac{1}{2} \alpha Q(x_1, x_2; K_D) \\ -\frac{1}{2} \alpha Q(x_1, x_2; K_D)^T & K_D - \alpha M(x_1) \end{bmatrix}$$

and  $K_D - \alpha M(x_1)$  be positive definite by choosing  $K_P > 0$  and  $K_D > 0$  large enough. In other words, if  $K_P > 0$  and  $K_D > 0$  are large enough and  $K_I > 0$  is small, there exists  $\alpha$  such that the above matrix and  $K_D - \alpha M(x_1)$  become positive definite for the given  $\beta$ .

Let  $\Omega_c = \{(x, z) \mid V(x, z) \leq c\}$  and suppose that  $\Omega_c$  is bounded and  $\dot{V}(x, z) \leq 0$  in  $\Omega_c$  ( $c$  is a positive number such that  $\dot{V}(x, z) \leq 0$ ). Here define  $\Omega_E$  as a set of all points of  $\Omega_c$  satisfying  $\dot{V}(x, z) = 0$  and put

$$\Omega_E = \{(x, z) \mid \dot{V}(x, z) = 0, (x, z) \in \Omega_c\} \tag{41}$$

From (40),(2),(36)  $(x, z)$  satisfying  $\dot{V}(x, z) = 0$  is given as  $x_1^* - x_1 = 0$ ,  $x_2 = 0$ ,  $z = \bar{z}$ , that is to say, it is a point  $(x_1, x_2, z) = (x_1^*, 0, \bar{z})$ . Accordingly, we know from (40),(2),(36),(37) that  $(x, z)$  in  $\Omega_E$  consists of only the equilibrium point  $(x_{1e}, x_{2e}, z_e) = (x_1^*, 0, \bar{z})$  when  $\tau = K_I \bar{z} = g(x_1^*)$ . Thus, the largest invariance set  $\Omega_M$  in  $\Omega_E$  consists of only the equilibrium point  $(x_1^*, 0, \bar{z})$ . Therefore, by LaSalle's invariance principle all trajectories in  $\Omega_c$  converges to  $\Omega_M$ , i.e., to  $(x_1^*, 0, \bar{z})$  as  $t \rightarrow \infty$ . Thus  $x = (x_1^*, 0)$  is asymptotically stable. Q.E.D



## **Robot Manipulators New Achievements**

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Robot manipulators are developing more in the direction of industrial robots than of human workers. Recently, the applications of robot manipulators are spreading their focus, for example Da Vinci as a medical robot, ASIMO as a humanoid robot and so on. There are many research topics within the field of robot manipulators, e.g. motion planning, cooperation with a human, and fusion with external sensors like vision, haptic and force, etc. Moreover, these include both technical problems in the industry and theoretical problems in the academic fields. This book is a collection of papers presenting the latest research issues from around the world.

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