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Fuzzy Optimal Control for Robot Manipulators

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1. Introduction

This chapter deals with the design of a Fuzzy Logic Controller based Optimal Linear Quadratic Regulator (FC-LQR) for the control of a robotic system. The main idea is to design a supervisory fuzzy controller capable to adjust the controller parameters in order to obtain the desired axes positions under variations of the robot parameters and payload variations.
In the advanced control of robotic manipulators, it is important for manipulators to track trajectories in a wide range of work place. If speed and accuracy is required, the control using conventional methods is difficult to realize because of the high nonlinearity of the robot system.
In control design, it is often of interest to design a controller to fulfil, in an optimal form, certain performance criteria and constraints in addition to stability. The theme of optimal control addresses this aspect of control system design. For linear systems, the problem of designing optimal controllers reduces to solving algebraic Riccati equations , which are usually easy to solve and detailed literature of their solutions can be found in many references . Nevertheless, for nonlinear systems, the optimization problem reduces to the so-called Hamilton-Jacobi (HJ) equations, which are nonlinear partial differential equations. Different from their counterparts for linear systems, HJ equations are usually difficult to solve both numerically and analytically. Improvements have also been carried out on the numerical solution of the approximated solution of HJ equations. But few results so far can provide an effective way of designing optimal controllers for general nonlinear systems.
In the past, the design of controllers based on a linearized model of real control systems. In many cases a good response of complex and highly non-linear real process is difficult to obtain by applying conventional control techniques which often employ linear mathematical models of the process. One reason for this lack of a satisfactory performance is the fact that linearization of a non-linear system might be valid only as an approximation to the real system around a determined operating point.
However, fuzzy controllers are basically non-linear, and effective enough to provide the desired non-linear control actions by carefully adjusting their parameters.
In this chapter, we propose an effective method to nonlinear optimal control based on fuzzy control. The optimal fuzzy controller is designed by solving a minimization problem that minimizes a given quadratic performance function.
Both the controlled system and the fuzzy controller are represented by the affine Takagi-Sugeno (T-S) fuzzy model taking into consideration the effect of the constant term. Most of the research works analyzed the T-S model assuming that the non-linear system is linearized

with respect to the origin in each IF-THEN rule (Tanaka and Sano 1994), (Tanaka et al. 1996), which means that the consequent part of each rule is a linear function with zero constant term. This will in turn reduce the accuracy of approximating non-linear systems. Moreover, in linear control theory, the independent term does not affect the dynamics of the system rather the input to it. In the case of fuzzy control, the fuzzy system is resulted from blending all the subsystems. The blending of the independent term of each rule will no longer be a constant but a function of the variables of the system and thus affects the dynamics of the resultant system. A necessary condition has been added to deal with the independent term. The final fuzzy system can be obtained by blending of these affine models. The control is carried out based on the fuzzy model via the so-called parallel distributed compensation scheme. The idea is that for each local affine model, an affine linear feedback control is designed. The resulting overall controller, which is also a non-linear one, is again a blending of each individual affine linear controller.

LQR is used to determine best values for parameters in fuzzy control rules in which the robustness is inherent in the LQR thereby robustness in fuzzy control can be improved. With the aid of LQR, it provides an effective design method of fuzzy control to ensure robustness.

In this chapter, we will show how the LQR, the structure of which is based on mathematical analysis, can be made more appropriate for actual implementation by introduction of fuzzy rules.

The motivation behind this scheme is to combine the best features of fuzzy control and LQR to achieve rapid and accurate tracking control of a class of nonlinear systems.

The results obtained show a robust and stable behavior when the system is subjected to various initial conditions, moment of inertia and to disturbances.

The content of this chapter is organized as follows. In section 2, an Overview of various control techniques for robot manipulators are presented. Section 3 presents the modelling of the robot manipulator. Section 4 demonstrates Takagi-Sugeno model for the robot manipulator under study. In section 5 a detailed mathematical description of the proposed optimal controller is presented. Section 6 entails the application of the proposed FC-LQR on a robot manipulator to demonstrate the validity of the proposed approach. This example shows that the proposed approach gives a stable and well damped response infront of various initial conditions, moment of inertia and a robust behaviour in the presence of disturbances. The conclusion of the effectiveness and validity of the proposed apprach is explained in section 7.

2. Overview of Control Techniques for Robot Manipulators

It is well known that robotic manipulators are complicated, dynamically coupled, highly time-varying, highly nonlinear systems that are extensively used in tasks such as welding, paint spraying, accurate positioning systems and so on. In these tasks, end-effectors of robotic manipulators are commended to move from one place to another, or to follow some given trajectories as close as possible. Therefore, trajectory tracking problem is the most significant and fundamental task in control of robotic manipulators.

Motivated by requirements such as a high degree of automation and fast speed operation from industry, in the past decades, various control methods are introduced in the publications such as proportional, integration, derivative (PID) control (Luh 1983), feed-forward compensation control (Khosla and Kanade 1988), adaptive control (Slotine and Li 1988), variable structure control (Slotine et al. 1983), neural networks control (Purwar et al. 2005), fuzzy control (Chen et al. 1998) and so on.

As a predominant method in industrial robotic manipulators, traditional PID control has simple structure and convenient implementation (Luh 1983). However, some strong assumptions are required to be made, which involve that each joint of robotic manipulators is decoupled from others and the system has to be in the status of slow motion. Control performance degrades quickly as operating speed increases. Therefore, a robotic manipulator controlled in this way is only appropriate for relatively slow motion.

Robotic manipulator systems are inevitably subject to structured and unstructured uncertainty. Structured uncertainty is characterized by a correct dynamical model with parameters variations, which results from difference in weights, sizes and mass distributions of payloads manipulated by robotic manipulators, difference in links properties of robotic manipulators, difference in inaccuracies on torque constants of actuators and so on. Unstructured uncertainty is characterized by unmodeled dynamics, which is due to the presence of external disturbances, high-frequency modes ofrobotic manipulators, neglected time-delays and nonlinear frictions and so on.

Structured uncertainty can result in imprecision of dynamical models of robotic manipulators, and controllers designed for nominal parameters may not properly work for all changes in parameters. Adaptive control techniques (Slotine and Li 1988), can be used in this case. However, adaptive control law is unable to handle unstructured uncertainty. To overcome this difficulty, variable structure control (Slotine et al. 1983) that can simultaneously attenuate influences ofboth structured and unstructured uncertainty is employed. Unfortunately, undesirable chattering on sliding surface due to high frequent switching can deteriorate system performances, which cannot be eliminated completely.

For practical and complex control problem of robotic manipulators, traditional and effective schemes also cannot be ignored. Computed Torque Control (CTC) (Middleton and Goodwin 1988) is worth noting, because CTC is easily understood and of good performances. Briefly speaking, CTC is a linear control method to linearize and decouple robotic dynamics by using perfect dynamical models of robotic manipulator systems in order that motion ofeach joint can be individually controlled using other well-developed linear control strategies.

However, CTC method for robotic manipulators suffers from two difficulties. First, CTC requires exact dynamical knowledge of robotic manipulators, which is apparently impossible in practical situations. Second, CTC is not robust to structured uncertainty and/or unstructured uncertainty, which may result in performance devaluation.

One of successful fuzzy systems' (FS) applications is to model complex nonlinear systems by a set of fuzzy rules. One important property of fuzzy modeling approaches is that FS is a universal approximator (Wang and mendel 1992). In other words, FS can approximate virtually any nonlinear functions to arbitrary accuracy provided that enough rules are given. FS for control, i.e. Fuzzy Controller (FC) can integrate expertise of skilled personnel into control procedure and mathematical model is not required. Over the last few years, FC for complex nonlinear systems have been developed extensively (Hua et al. 2004), (Kim and Lewis 1999).

Recently, much attention has been devoted to FC for robotic manipulators. The latest survey on FC for robotic manipulators can be found in (Purwar et al. 2005) and references cited therein. Sun (Luh 1983) combined FC and variable structure control to construct a controller, where FS was greatly simplified by using system representative point and its derivative as inputs. Control laws designed by Hsu (Sun et al. 1999) consisted of a regular fuzzy controller and a supervisory control term, which ensured stability of closed-loop systems. In (Labiod et al. 2005), two FC schemes for a class of uncertain continuous-time multi-input

multi-output nonlinear dynamical systems were derived. Satisfactory performances were achieved by applying them to robotic manipulators (Song et al. 2006).
In (Song et al. 2006), it is supposed that robotic manipulator systems with structured uncertainty and/or unstructured uncertainty can be separated as two subsystems: nominal system with precise dynamical knowledge and uncertain system with unknown knowledge. An approach of CTC plus FC compensator is proposed.
The nominal system is controlled using CTC and for uncertain system, a fuzzy controller is designed. Here the fuzzy controller acts as compensator for CTC. Parameters updating laws of the fuzzy controller are derived using Lyapunov stability theorem.
FS have also been extensively adopted in adaptive control of robot manipulators (Berstecher et al. 2001), (Chuan-Kai Lin 2003), (Li et al. 2001), (Tzes et al. 1993), (Tong et al. 2000), (Tsai et al. 2000), (Yi and Chung 1997), (Yoo and Ham 2000), (Zhou et al. 1992), (Fukuda et al. 1992), (Meslin et al. 1992), (Sylvia et al. 2003).
In (Berstecher et al. 2001), Berstecher develops a linguistic heuristic-based adaptation algorithm for a fuzzy sliding mode controller. The algorithm relies on the linguistic knowledge in the form of fuzzy IF-THEN rules. Tsai et al. (Tsai et al. 2000) propose a robust multilayer fuzzy controller for the model following control of robot manipulators with torque disturbance and measurement noise.
Yi and Chung (Yi and Chung 1997) define a set of fuzzy rules based on the knowledge of error and derivative of error for designing the controller. Yoo and Ham (Yoo and Ham 2000) exploit the function approximation capabilities of FS to compensate for the parametric uncertainties of the robot manipulator. Chuan-Kai Lin (Chuan-Kai Lin 2003) proposes reinforcement learning systems combined with fuzzy control for robot arms. Here the reinforcement learning signal is used to update the weights of a fuzzy logic system which is used to approximate an unknown nonlinear function. This approximated function is then used for computing the control law. In (Li et al. 2001) Li presents a hybrid control scheme for tracking control of a manipulator which consists of a fuzzy logic proportional controller and a conventional integral and derivative controller.
Moreover, this controller was compared to a conventional PID controller and the performance of the fuzzy P+ID controller was found superior to conventional PID controller. In (Sylvia et al. 2003) Sylvia Kohn-Rich and Henryk Flashner present tracking control problem of mechanical systems based on Lyapunov stability theory and robust control of nonlinear systems. The control law has a two-component structure conventional PD control and a fuzzy component of robust control which is aimed at minimizing the chattering effect. Tong Shaocheng et al. (Tong et al. 2000) develops a robust fuzzy adaptive controller for a class of unknown nonlinear systems. In the control procedure, FS are implemented to estimate the unknown functions and robust compensators are designed in H_∞ sense for attenuating the unmatched uncertainties. In (Zhang et al. 2000), Rainer palm develops a mamdani fuzzy controller following the pattern of suboptimal control. The proposed controller in the paper is compared and found to have higher tracking quality than a conventional PD controller. In (Fuchun et al. 2003), Fuchun Sun et al. propose a nuero fuzzy adaptive control methodology for trajectory tracking of robotic manipulators. Here the fuzzy dynamic model of the manipulator is established using the Tagaki-Sugeno fuzzy framework. Based on the derived fuzzy dynamics of the manipulator, the neuro fuzzy adaptive controller is developed to improve the system performance by adaptively modifying the fuzzy model parameters. All these methods require both the position and velocity measurements, which can be problematic in practice (Purwar et al. 2005).

Applications in tracking control problems of robot manipulators are also available (Commuri et al. 1996), (Jagannathan et al. 1996), (Llama et al. 1998).
In (Commuri et al. 1996) an adaptive fuzzy logic controller is proposed. The structure of this controller is based on the so-called SlotineŰ Li controller (a PD term plus a model-based nonlinear compensation term using Hltered tracking errors). A framework that can approximate any nonlinear function with arbitrary accuracy is designed using a fuzzy logic system. By using this technique an estimate of the nonlinear compensation term of the control law is obtained. A learning algorithm that learns the membership function is developed, and the stability of the closed-loop system is demonstrated. In (Jagannathan et al. 1996) a tracking control system of a class of feedback linearizable unknown nonlinear dynamical systems, such as robotic systems, using a discrete time fuzzy logic controller, is presented.
Unlike (Commuri et al. 1996), instead of using fuzzy adaptation of the nonlinear compensation terms, in this paper the potential of a gain scheduling fuzzy self-tuning scheme is used in order to design a methodology for on-line parameter tuning of a robot motion controller. Particular attention is paid to provide a rigorous stability analysis including the robot nonlinear dynamics.
A basic problem in controlling robots is the so-called motion control formulation where a manipulator is requested to track a desired position trajectory. A number of such robot motion controllers having rigorous stability proofs have been reported in the literature and robotics textbooks (Lewis et al. 1994), (Sciavicco et al. 1996). Most of these stability results have been obtained provided that the controller parameters are constant and they belong to well-defined intervals (Llama et al. 2001).
In (Purwar et al. 2005), a stable fuzzy adaptive controller for trajectory tracking is developed for robot manipulators without velocity measurements, taking into account the actuator constraints. The controller is based on structural knowledge of the dynamics of the robot and measurements of link positions only. The gravity torque including system uncertainty like payload variation, etc., is estimated by FS. The proposed controller ensures the local asymptotic stability and the convergence of the position error to zero. The proposed controller is robust not only to structured uncertainty such as payload parameter variation, but also to unstructured one such as disturbances. The validity of the control scheme is shown by simulations on a two-link robot manipulator.
In (Llama et al. 2001) a motion control scheme based on a gain scheduling fuzzy self-tuning structure for robot manipulators is presented. They demonstrate, by taking into account the full non-linear and multivariable nature of the robot dynamics, that the overall closed-loop system is uniformly asymptotically stable. Besides the theoretical result, the proposed control scheme shows two practical characteristics. First, the actuators torque capabilities can be taken into account to avoid torque saturation, and second, undesirable e8ects due to Coulomb friction in the robot joints can be attenuated. Experimental results on a two degrees-of-freedom direct-drive arm show the usefulness of the proposed control approach.

3. Modelling of Robot Manipulators

The robot under study is characterized by having six rotational joints driven by hydraulic actuators(motors for the first joint and the robot wrist, and cylinders for other axes).
The main problem in controlling such processes is the nonlinearity. This makes it very difficult the use of conventional control techniques to implement the control job.
In this chapter, the robot which is a highly non-linear system is represented by affine T-S model, where the consequent part of each rule represents an affine model of the original sys-

tem in a certain operating point. The final fuzzy system can be obtained by blending of these affine models. The control is carried out based on the fuzzy model via the so-called parallel distributed compensation scheme. The idea is that for each local affine model, an affine variable structure controller is designed. the resulting overall controller, which is also a non-linear one, is again a blending of each individual affine linear controller.
The behaviour of the robot depends upon the robot working conditions, in particular the axes positions and the payload which are considered as the premise part of the fuzzy rule (Purwar et al. 2005), (Song et al. 2006).
The suggested fuzzy control considers every axis as a system whose control variables has to be tuned. It is necessary to establish differences between the first axis, which implies a rotation in the horizontal plane, and the axes 2,3 and 4, which imply rotations in the vertical plane. In the case of the latter two axes, which drive the robot wrist, it is not necessary to adjust the control parameters in real time, and they are automatically adjusted when the robot payload changes. For the latter two axes, due to the short length of the driven links and the robot kinematic configuration, their angular position doesnot have a significant amount of influence on their dynamic behaviour, which is mainly determined by the payload. All this means that these two axes are considered independientes with repsect to their control and influence on the adjustment of the other previous axes.
The variables that define the behaviour of each one of the axes are the angular values in each joint and the extreme payload. We should mention that not all the robot joints will influence the dynamic behaviour. The first axis position does not influence the others.
The angular values of the vertical joints that are placed behined the joint we are considering along the robot kinematic chain, and which influence the dynamic behaviour, can be combined in one fuzzy variable. Denoting the angular value for the joint j by θ_j, the effective angular value θ_{ia} to be considered as a fuzzy input variable for axes 2, 3 and 4 is:

$$\theta_{ia} = \sum_{j=2}^{i} \theta_j, \quad i = 2, 3, 4$$

Similarly, considering one particular axis, the angular axis, the angular values of the vertical joints that ar placed in front of it, as well as the robot payload, can be combined in the other fuzzy input variable, namely the effective moment of inertia from the considered axis J_i. This can be represented as:

$$J_i = f(\theta_{j>i}, M_{j>i}, M)$$

Where

- J_i represents the effective moment of inertia from axis i
- $\theta_{j>i}$ represents the angular values of the axes after i
- M_j represents the mass of the link j including its actuator
- M represents the mass of payload.

Figure 1 shows the scheme for the fuzzy input variable for axes 2,3 and 4.

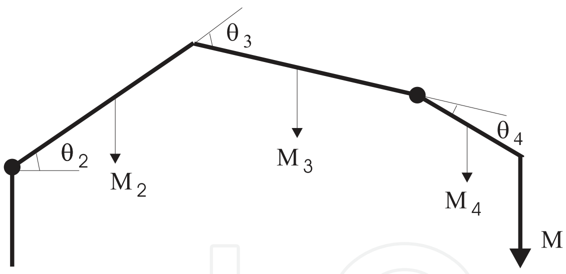

Fig. 1. scheme for the fuzzy input variable for axes 2,3 and 4

4. Takagi-Sugeno Model of Robot Manipulators

Consider the following system:

$$\acute{x} = f(x, u)$$

where

$$x = (x_1, x_2, \ldots, x_n)^t$$
$$u = (u_1, u_2, \ldots, u_m)^t$$

The local dynamics in various equilibrium states are represented by affine subsystems as follows:
Both the fuzzy system and the fuzzy controller are represented by the affine T-S fuzzy model. Let the $(\mathrm{i}_1 \ldots \mathrm{i}_\mathrm{n})^{\mathrm{th}}$ rule of the T-S model be represented as:

$$\begin{aligned} S^{(i_1 \ldots i_n)} &: \textit{If } x \textit{ is } M_1^{i_1} \textit{ and } \acute{x} \textit{ is } M_2^{i_2} \textit{ and} \ldots \\ &\textit{and } x^{(n-1)} \textit{ is } M_n^{i_n} \textit{ then} \\ \acute{x} &= a_0^{(i_1 \ldots i_n)} + A^{(i_1 \ldots i_n)}\mathbf{x} + b^{(i_1 \ldots i_n)} u \end{aligned} \tag{1}$$

where $\mathrm{M}_1^{\mathrm{i}_1}$ $(\mathrm{i}_1 = 1, 2, \ldots, \mathrm{r}_1)$ are fuzzy sets for $\mathbf{x}$, $\mathrm{M}_2^{\mathrm{i}_2}$ $(\mathrm{i}_2 = 1, 2, \ldots, \mathrm{r}_2)$ are fuzzy sets for $\dot{\mathrm{x}}$, $\mathrm{M}_\mathrm{n}^{\mathrm{i}_\mathrm{n}}$ $(\mathrm{i}_\mathrm{n} = 1, 2, \ldots, \mathrm{r}_\mathrm{n})$ are fuzzy sets for $\mathrm{x}^{(\mathrm{n}-1)}$. Therefore the complete fuzzy system has $\mathrm{r}_1 \times \mathrm{r}_2 \times \ldots \mathrm{r}_\mathrm{n}$ rules.
We will adapt the affine T-S model to our robotic system. The premise part of each rule depends on the effective angular value and the effective moment of inertia. Both of them are linearized in three operating points. Table 1 shows the variables of each rule of the robotic system represented by T-S model. The input fuzzy variable which represent the angular axis position is linearized in three operating points. The moment of inertia is linearized in three operating points (Ishikawa 1988). The results were obtained from several tens of experiments of the real system (Gamboa 1996). The system has been approximated in each operating point by a linearized mathematical model looking for a suitable model that coincides with the nonlinear system.
Figure 2 shows the following triangular fuzzy sets of the angular position of the second axis:

$$\begin{aligned} \theta_{2a}^1 &= \{-\infty, 0, 55\} \\ \theta_{2a}^2 &= \{0, 55, 115\} \\ \theta_{2a}^3 &= \{55, 115, \infty\} \end{aligned} \tag{2}$$

Variable	Universe	Label
θ_{2a}	$[0°, 115°]$	$\{M_{\theta 2}^1, M_{\theta 2}^2, M_{\theta 2}^3\}$
θ_{3a}	$[-120°, 90°]$	$\{M_{\theta 3}^1, M_{\theta 3}^2, M_{\theta 3}^3\}$
θ_{3a}	$[-240°, 90°]$	$\{M_{\theta 4}^1, M_{\theta 4}^2, M_{\theta 4}^3\}$
J_2	$[5000, 51540]$	$\{M_{J2}^1, M_{J2}^2, M_{J2}^3\}$
J_3	$[1500, 18564]$	$\{M_{J3}^1, M_{J3}^2, M_{J3}^3\}$
J_4	$[140, 5093]$	$\{M_{J4}^1, M_{J4}^2, M_{J4}^3\}$

Table 1. Input fuzzy variables

$$
\begin{aligned}
\theta_{2a}^1 &= \{-\infty \ \ , 0, 55\} \\
\theta_{2a}^2 &= \{0, 55, 115\} \\
\theta_{2a}^3 &= \{55, 115, \infty\}
\end{aligned}
$$

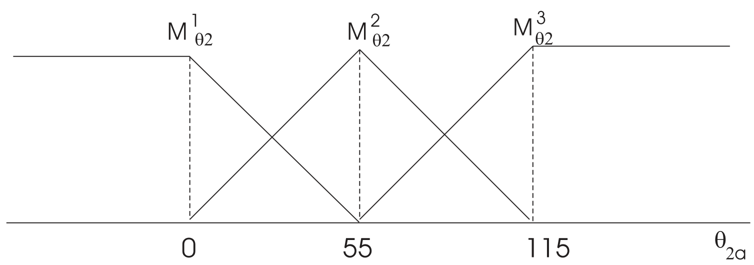

Fig. 2. Fuzzy sets of angular position of the second axis

Figure 3 shows the following triangular fuzzy sets of the moment of inertia of the second axis:

$$
\begin{aligned}
J_{2a}^1 &= \{-\infty, 5000, 25000\} \\
J_{2a}^2 &= \{5000, 25000, 51540\} \\
J_{2a}^3 &= \{25000, 51540, \infty\}
\end{aligned} \tag{3}
$$

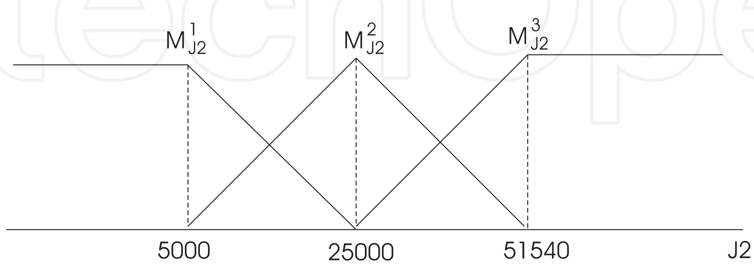

Fig. 3. Fuzzy sets of the moment of inertia of the second axis

Firstly, The model of the robotic model is linearized in three operation points for both the angular postion and its moment of interita. The universe of discourse of the anglular position is $[0, 115]$ rad. and the one of the moment of inertia is $[5000, 51540]$. The resultant identified fuzzy system is described as follows:

$$
\begin{aligned}
&S_2^{11} : If \ \left(\theta_{2a} \ is \ M_{\theta 2}^1\right) \ and \ (\ J_2 \ is \ M_{J2}^1 \ ) \ then \\
&\ddot{\theta}_{2a}(t) = -77.4\dot{\theta}_{2a}(t) - 3947.5\theta_{2a}(t) + 66150u(t) \\
&S_2^{12} : If \ \left(\theta_{2a} \ is \ M_{\theta 2}^1\right) \ and \ (\ J_2 \ is \ M_{J2}^2 \ ) \ then \\
&\ddot{\theta}_{2a}(t) = -43.8\dot{\theta}_{2a}(t) - 3276.4\theta_{2a}(t) + 48391u(t) \\
&S_2^{13} : If \ \left(\theta_{2a} \ is \ M_{\theta 2}^1\right) \ and \ (\ J_2 \ is \ M_{J2}^3 \ ) \ then \\
&\ddot{\theta}_{2a}(t) = -49.2\dot{\theta}_{2a}(t) - 1754.5\theta_{2a}(t) + 24964u(t) \\
&S_2^{21} : If \ \left(\theta_{2a} \ is \ M_{\theta 2}^2\right) \ and \ (\ J_2 \ is \ M_{J2}^1 \ ) \ then \\
&\ddot{\theta}_{2a}(t) = -74.4\dot{\theta}_{2a}(t) - 3452.4\theta_{2a}(t) + 59525u(t) \\
&S_2^{22} : If \ \left(\theta_{2a} \ is \ M_{\theta 2}^2\right) \ and \ (\ J_2 \ is \ M_{J2}^2 \ ) \ then \\
&\ddot{\theta}_{2a}(t) = -41.7\dot{\theta}_{2a}(t) - 3007.6\theta_{2a}(t) + 1.65 + 45907u(t) \\
&S_2^{23} : If \ \left(\theta_{2a} \ is \ M_{\theta 2}^2\right) \ and \ (\ J_2 \ is \ M_{J2}^3 \ ) \ then \\
&\ddot{\theta}_{2a}(t) = -51.1\dot{\theta}_{2a}(t) - 1832.8\theta_{2a}(t) + 3.3 + 26471u(t) \\
&S_2^{31} : If \ \left(\theta_{2a} \ is \ M_{\theta 2}^3\right) \ and \ (\ J_2 \ is \ M_{J2}^1 \ ) \ then \\
&\ddot{\theta}_{2a}(t) = -74.1\dot{\theta}_{2a}(t) - 3540.3\theta_{2a}(t) + 63995u(t) \\
&S_2^{32} : If \ \left(\theta_{2a} \ is \ M_{\theta 2}^3\right) \ and \ (\ J_2 \ is \ M_{J2}^2 \ ) \ then \\
&\ddot{\theta}_{2a}(t) = -33.4\dot{\theta}_{2a}(t) - 2379\theta_{2a}(t) + 11.74 + 39647u(t) \\
&S_2^{33} : If \ \left(\theta_{2a} \ is \ M_{\theta 2}^3\right) \ and \ (\ J_2 \ is \ M_{J2}^3 \ ) \ then \\
&\ddot{\theta}_{2a}(t) = -50.7\dot{\theta}_{2a}(t) - 1777.6\theta_{2a}(t) + 23.43 + 28130u(t)
\end{aligned} \tag{4}
$$

5. Design of an Optimal Controller

In this section, a design of a fuzzy optimal controller based on linear quadratic regulator is carried out for a robotic manipulator whose model can be described in the following form:

$$x^{(n)} = f(x, \acute{x}, \ldots, x^{(n-1)}, u)$$

The T-S model can be adjusted as follows:
The IF-THEN rules are as follows:

$$
\begin{aligned}
&S^{(i_1 \cdots\cdots i_n)} : If \ x \ is \ M_1^{i_1} \ and \ \acute{x} \ is \ M_2^{i_2} \ and \ldots \ and \ x^{(n-1)} \ \ is \ \ M_n^{i_n} \\
&then \ \ x^{(n)} = a_o^{(i_1 \ldots i_n)} + a_1^{(i_1 \ldots i_n)} x + a_2^{(i_1 \ldots i_n)} \acute{x} + \ldots + a_n^{(i_1 \ldots i_n)} x^{(n-1)} + b^{(i_1 \ldots i_n)} u
\end{aligned} \tag{5}
$$

where $M_1^{i_1}$ $(i_1 = 1, 2, \ldots, r_1)$ are fuzzy sets for x, $M_2^{i_2}$ $(i_2 = 1, 2, \ldots, r_2)$ are fuzzy sets for $\acute{x}$ and $M_n^{i_n}$ $(i_n = 1, 2, \ldots, r_n)$ are fuzzy sets for $x^{(n-1)}$.

The fuzzy system is described as:

$$x^{(n)} = \frac{\sum_{i_1=1}^{r_1} \cdots \sum_{i_n=1}^{r_n} w^{(i_1 \ldots i_n)}(x) \left[a_0^{(i_1 \ldots i_n)} + a_1^{(i_1 \ldots i_n)} x \right]}{\sum_{i_1=1}^{r_1} \cdots \sum_{i_n=1}^{r_n} w^{(i_1 \ldots i_n)}(x)} + \frac{\sum_{i_1=1}^{r_1} \cdots \sum_{i_n=1}^{r_n} w^{(i_1 \ldots i_n)}(x) \left[a_2^{(i_1 \ldots i_n)} \acute{x} + a_n^{(i_1 \ldots i_n)} x^{(n-1)} + b^{(i_1 \ldots i_n)} u \right]}{\sum_{i_1=1}^{r_1} \cdots \sum_{i_n=1}^{r_n} w^{(i_1 \ldots i_n)}(x)} \tag{6}$$

The controller fuzzy rule is represented in a similar form:

$$\begin{gathered} C^{(i_1 \cdots \cdots i_n)} : \textit{If } x \textit{ is } M_1^{i_1} \textit{ and } \acute{x} \textit{ is } M_2^{i_2} \textit{ and} \ldots \textit{ and } x^{(n-1)} \textit{ is } M_n^{i_n} \\ \textit{then } u = k_r^{(i_1 \ldots i_n)} r - (k_0^{(i_1 \ldots i_n)} + k_1^{(i_1 \ldots i_n)} x + k_2^{(i_1 \ldots i_n)} \acute{x} + \ldots + k_n^{(i_1 \ldots i_n)} x^{(n-1)}) \end{gathered} \tag{7}$$

The closed-loop system is obtained substituting (7) in (5) as follows:

$$\begin{gathered} SC^{(i_1 \cdots \cdots i_n)} : \textit{If } x \textit{ is } M_1^{i_1} \textit{ and } \acute{x} \textit{ is } M_2^{i_2} \textit{ and} \ldots \textit{ and } x^{(n-1)} \textit{ is } M_n^{i_n} \\ \textit{then } x^{(n)} = a_o^{(i_1 \ldots i_n)} + a_1^{(i_1 \ldots i_n)} x + \cdots + a_n^{(i_1 \ldots i_n)} x^{(n-1)} + \\ b^{(i_1 \ldots i_n)} [k_r^{(i_1 \ldots i_n)} r - (k_0^{(i_1 \ldots i_n)} + k_1^{(i_1 \ldots i_n)} x + k_2^{(i_1 \ldots i_n)} \acute{x} + k_n^{(i_1 \ldots i_n)} x^{(n-1)})] \end{gathered} \tag{8}$$

5.1 Calculation of the Affine Term

The proposed methodology of design is based on the possibility of formulate the feedback system as shown previously in (8),
The affine term of the control action is used to eliminate the affine term of the system:

$$a_o^{(i_1 \ldots i_n)} + b^{(i_1 \ldots i_n)} k_0^{(i_1 \ldots i_n)} = 0$$

$$k_0^{(i_1 \ldots i_n)} = \frac{-a_o^{(i_1 \ldots i_n)}}{b^{(i_1 \ldots i_n)}}$$

and the feedback system is rewritten as follows:

$$\begin{gathered} SC^{(i_1 \cdots \cdots i_n)} : \textit{If } x \textit{ is } M_1^{i_1} \textit{ and } \acute{x} \textit{ is } M_2^{i_2} \textit{ and} \ldots \textit{ and } x^{(n-1)} \textit{ is } M_n^{i_n} \\ \textit{then } x^{(n)} = a_1^{(i_1 \ldots i_n)} x + \ldots + a_n^{(i_1 \ldots i_n)} x^{(n-1)} + \\ b^{(i_1 \ldots i_n)} [k_r^{(i_1 \ldots i_n)} r - k_1^{(i_1 \ldots i_n)} x + k_2^{(i_1 \ldots i_n)} \acute{x} + \ldots + k_n^{(i_1 \ldots i_n)} x^{(n-1)})] \end{gathered} \tag{9}$$

5.2 State Space Feedback Control based Linear Quadratic Regulator

Any control methodology by state feedback design can be applied to calculate the rest of control coefficients as pole assignments for example. The well known Linear Quadratic Regulator (LQR) method might be an appropriate choice. The system can be represented in state space form:

$$\acute{x} = Ax + Bu$$

$$x \in \Re^n, u \in \Re^m, A \in \Re^{n\times n}, B \in \Re^{n\times m}$$

The objective is to find the control action u(t) to transfer the system from any initial state $x(t_0)$ to some final state $x(\infty) = 0$ in an infinite time interval, minimizing a quadratic performance index of the form:

$$J = \int_{t_0}^{\infty} (x^t Qx + u^t Ru)dt$$

where $Q \in \Re^{n\times n}$ is a symmetric matrix, at least positive a semidefinite one and $R \in \Re^{m\times m}$ is also a symetric positive definite matrix and K is referred to as the state feedback gain matrix. The optimal control law is then computed as follows:

$$u(t) = -Kx(t) = -R^{-1}B^t Lx(t) \tag{10}$$

where the matrix $L \in \Re^{n\times n}$ is a solution of the Riccati equation:

$$0 = -Q + LBR^{-1}B^t L - LA - A^t L$$

The objective can be generalized to find the control action u(t) to transfer the system from any initial state $x(t_0)$ to any reference state $x(\infty) = x_r$ in an infinite time interval, minimizing a quadratic performance index of the form:

$$J = \int_{t_0}^{\infty} ((x - x_r)^t Q(x - x_r) + (u - u_r)^t R(u - u_r))dt$$

where u_r is the necessary input required to keep the system stable in the equilibrium state x_r, which can be calculated as follows:

$$0 = Ax_r + Bu_r \Longrightarrow u_r = -B^+ Ax_r$$

where B^+ is the pseudo inverse of B.
The solution in this case is:

$$u(t) - u_r = -K(x(t) - x_r) = -R^{-1}B^t L(x(t) - x_r) \tag{11}$$

$$u(t) = (K - B^+ A)x_r - Kx(t) \tag{12}$$

where L is the solution of the previously mentioned Riccati equation. Figure 4 shows a block diagram of the proposed optimal controller.
The design algorithm includes firstly the cancelation of the affine term in each subsystem of the form:

$$x^{(n)} = a_0^{(i_1 \dots i_n)} + a_1^{(i_1 \dots i_n)} x + a_2^{(i_1 \dots i_n)} \acute{x} + \dots + a_n^{(i_1 \dots i_n)} x^{(n-1)} + b^{(i_1 \dots i_n)} u \tag{13}$$

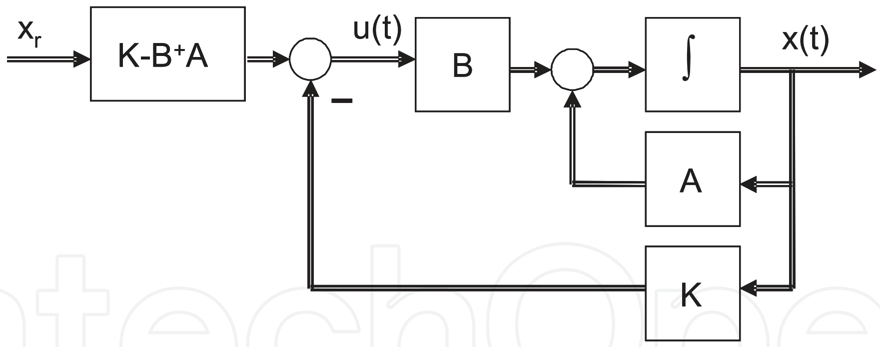

Fig. 4. A block diagram of the proposed optimal controller

The system is then represented in state space form as:

$$A^{(i_1 \dots i_n)} == \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \dots & 0 & 1 \\ a_1^{(i_1 \dots i_n)} & a_2^{(i_1 \dots i_n)} & a_3^{(i_1 \dots i_n)} & \dots & a_n^{(i_1 \dots i_n)} \end{bmatrix}, \; B^{(i_1 \dots i_n)} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ b^{(i_1 \dots i_n)} \end{bmatrix},$$

$$x = \begin{bmatrix} x & \acute{x} & \dots & x^{n-1} \end{bmatrix}^t$$

$$x_r = \begin{bmatrix} r & 0 & \dots & 0 \end{bmatrix}^t$$

Secondly, the LQR methodology is applied for each subsystem using a common state weighting matrix Q and input matrix R for all the rules. Thus, Riccati equation is solved for each subsystem as follows:

$$0 = -Q + L^{(i_1 \dots i_n)} B^{(i_1 \dots i_n)} R^{-1} B^{(i_1 \dots i_n)^t} L^{(i_1 \dots i_n)} - L^{(i_1 \dots i_n)} A^{(i_1 \dots i_n)} - A^{(i_1 \dots i_n)^t} L^{(i_1 \dots i_n)}$$

Then the the state feedback gain vector can be obtained from (10):

$$K^{(i_1 \dots i_n)} = \begin{bmatrix} k_1^{(i_1 \dots i_n)} & k_2^{(i_1 \dots i_n)} & \dots & k_n^{(i_1 \dots i_n)} \end{bmatrix} = R^{-1} B^{(i_1 \dots i_n)^t} L^{(i_1 \dots i_n)}$$

and finally,

$$u(t) = (K^{(i_1 \dots i_n)} - B^{(i_1 \dots i_n)^+} A^{(i_1 \dots i_n)}) x_r - K^{(i_1 \dots i_n)} x(t)$$

6. Application of the Proposed FC-LQR for Robotic Manipulator

A FC-LQR is designed which meets the requirements of small overshoot in the transient response and a well damped oscillations with no steady state error.
For example, in the first rule of the robot model described in (4), we have:

$$S_2^{11} : If \; \left(\theta_{2a} \; is \; M_{\theta 2}^1\right) \; and \; (\; J_2 \; is \; M_{J2}^1 \;) \; then$$
$$\ddot{\theta}_{2a}(t) = -77.4\dot{\theta}_{2a}(t) - 3947.5\theta_{2a}(t) + 66150u(t)$$

As the robot model in this rule has no affine term, there will be no affine term in the controller rule, this means that,

$$k_0^{11} = 0$$

and the state space model for this subsystem is:

$$A^{(11)} = \begin{bmatrix} 0 & 1 \\ -3947.5 & -77.4 \end{bmatrix}, \quad B^{(11)} = \begin{bmatrix} 0 \\ 66150 \end{bmatrix}$$

$$x = \begin{bmatrix} \theta_{2a} & \dot{\theta}_{2a} \end{bmatrix}^t$$

$$x_r = \begin{bmatrix} \theta_r & 0 \end{bmatrix}^t$$

If the weighting state and input matrices are:

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & 50 \end{bmatrix}, \quad R = \begin{bmatrix} 3.10^4 \end{bmatrix}$$

the resultant state feedback gain vectors are:

$$K^{(11)} = \begin{bmatrix} 0.2786.10^{-3} & 0.3967.10^{-2} \end{bmatrix}$$

$$K^{(11)} - B^{(11)+}A = \begin{bmatrix} 0.0600 & 0.0408 \end{bmatrix}$$

Thus, the control action can be calculated as follows:

$$u(t) = 0.0600\theta_r - 0.2786.10^{-3}\theta_{2a} - 0.3967.10^{-2}\dot{\theta}_{2a}$$

Following the same procedure, we can calculate the control action for the rest of the subsystems.

The design parameters in this case are Q and R matrices whose values can be adjusted by trial and error. The objective should be the adjustment of the system with sufficiently fast response under admissible control action u(t). Taking into consideration that the range of possible values for θ_{2a} is $0 \div 115$, while the range for the control action is $\pm 3\ V$, it seems reasonable weight the input signal more than the output. In fact, we found that the admissible results can be obtained for the input action are:

$$q_{11} = 1 \ \ R = [10^3]$$

and better results can be obtained with:

$$q_{11} = 1 \ \ R = [10^4]$$

With respect to the weighting of the angular velocity, it has been found that with $q_{22} = 1$, the response peaks approach $160°/s$ which is superior than the admissible range and with $q_{22} = 20$, the peaks are below $40°/s$ which are within the admissible range. To get the optimal response, we have chosen:

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & 20 \end{bmatrix}, \quad R = \begin{bmatrix} 10^4 \end{bmatrix}$$

and the control action for each subsystem is:

$$C_2^{11} : If\ \left(\theta_{2a}\ is\ M_{\theta 2}^1\right)\ and\ (\ J_2\ is\ M_{J2}^1\ )\ then$$
$$u(t) = 0.0605\theta_r - 0.8321.10^{-3}\theta_{2a} - 0.0436\dot{\theta}_{2a}$$
$$C_2^{12} : If\ \left(\theta_{2a}\ is\ M_{\theta 2}^1\right)\ and\ (\ J_2\ is\ M_{J2}^2\ )\ then$$
$$u(t) = 0.0684\theta_r - 0.7345.10^{-3}\theta_{2a} - 0.0438\dot{\theta}_{2a}$$
$$C_2^{13} : If\ \left(\theta_{2a}\ is\ M_{\theta 2}^1\right)\ and\ (\ J_2\ is\ M_{J2}^3\ )\ then$$
$$u(t) = 0.0710\theta_r - 0.7079.10^{-3}\theta_{2a} - 0.0428\dot{\theta}_{2a}$$
$$C_2^{21} : If\ \left(\theta_{2a}\ is\ M_{\theta 2}^2\right)\ and\ (\ J_2\ is\ M_{J2}^1\ )\ then$$
$$u(t) = 0.0589\theta_r - 0.8558.10^{-3}\theta_{2a} - 0.0434\dot{\theta}_{2a}$$
$$C_2^{22} : If\ \left(\theta_{2a}\ is\ M_{\theta 2}^2\right)\ and\ (\ J_2\ is\ M_{J2}^2\ )\ then$$
$$u(t) = 0.0663\theta_r - 0.0359.10^{-3} - 0.7588.10^{-3}\theta_{2a} - 0.0438\dot{\theta}_{2a}$$
$$C_2^{23} : If\ \left(\theta_{2a}\ is\ M_{\theta 2}^2\right)\ and\ (\ J_2\ is\ M_{J2}^3\ )\ then$$
$$u(t) = 0.0700\theta_r - 0.1247.10^{-3} - 0.7184.10^{-3}\theta_{2a} - 0.0428\dot{\theta}_{2a}$$
$$C_2^{31} : If\ \left(\theta_{2a}\ is\ M_{\theta 2}^3\right)\ and\ (\ J_2\ is\ M_{J2}^1\ )\ then$$
$$u(t) = 0.0562\theta_r - 0.8276.10^{-3}\theta_{2a} - 0.0436\dot{\theta}_{2a}$$
$$C_2^{32} : If\ \left(\theta_{2a}\ is\ M_{\theta 2}^3\right)\ and\ (\ J_2\ is\ M_{J2}^2\ )\ then$$
$$u(t) = 0.0608\theta_r - 0.2961.10^{-3} - 0.8276\theta_{2a} - 0.0439\dot{\theta}_{2a}$$
$$C_2^{33} : If\ \left(\theta_{2a}\ is\ M_{\theta 2}^3\right)\ and\ (\ J_2\ is\ M_{J2}^3\ )\ then$$
$$u(t) = 0.0640\theta_r - 0.8329 - 0.7863.10^{-3}\theta_{2a} - 0.0430\dot{\theta}_{2a}$$

Figure 5 shows the evolution of the angle θ_{2a} from an initial condition of 25° and zero reference signal It also shows the step response with reference input of 50° and a constant value of moment of inertia igual to $J_2 = 25000$. The step response has a settling time of 3 seconds.
Figure 6 shows the response with various initial conditions $10°, \ldots, 50°$ and zero reference input signal. After five seconds, the system is excited with various step reference inputs $10°, \ldots, 50°$ with a constant moment of inertia $J_2 = 25000$. It can be clearly observed that well damped and fast response is obtained in all the range of possible values of the output.

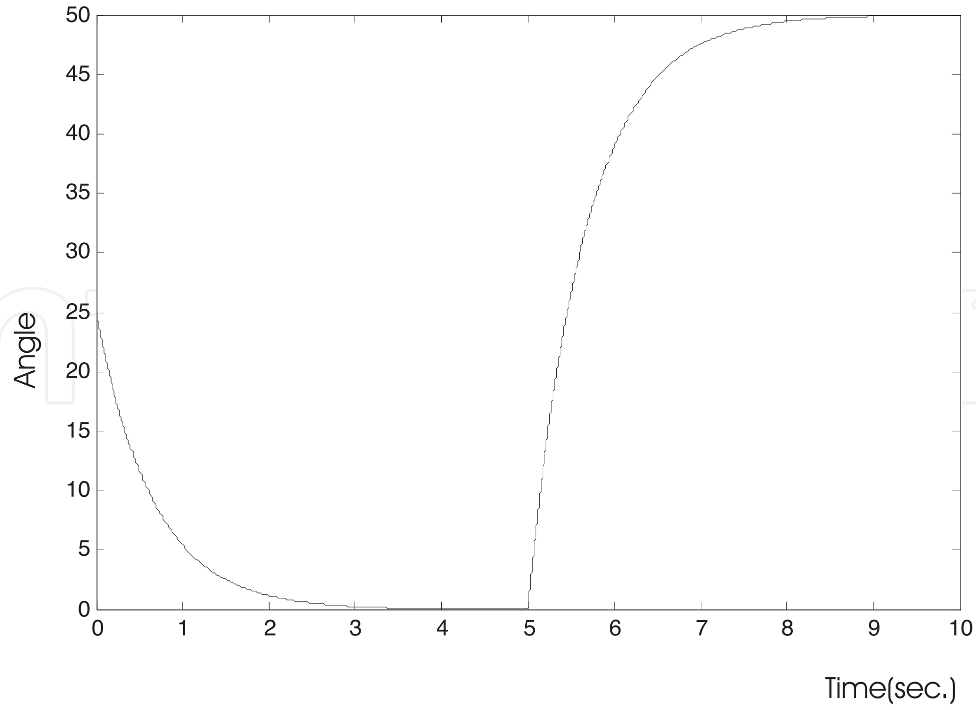

Fig. 5. Transient response of the robotic system with initial condition of 25° and moment of inertia $J_2 = 25000$

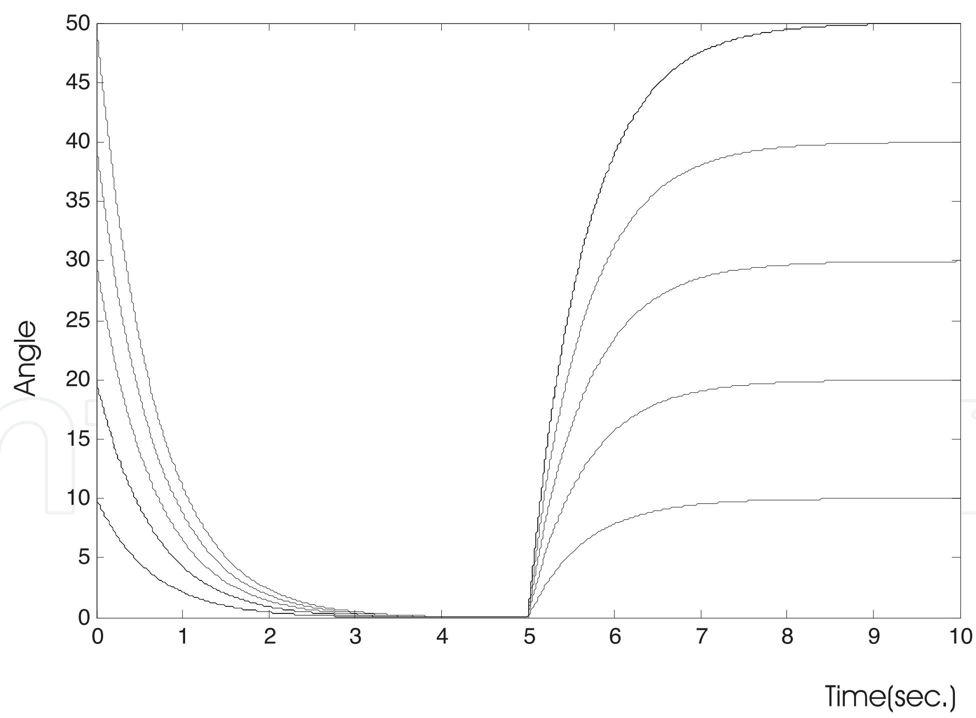

Fig. 6. Transient response of the robotic system with various initial conditions and reference input signals and constant moment of inertia of $J_2 = 25000$

Nevertheless, figure 7 shows the response with an intial condition and reference input signal of 25°. The response is initiated with moment of inertia $J_2 = 25000$ and after five seconds an abrupt change is applied in the moment of inertia to $J_2 = 50000$.

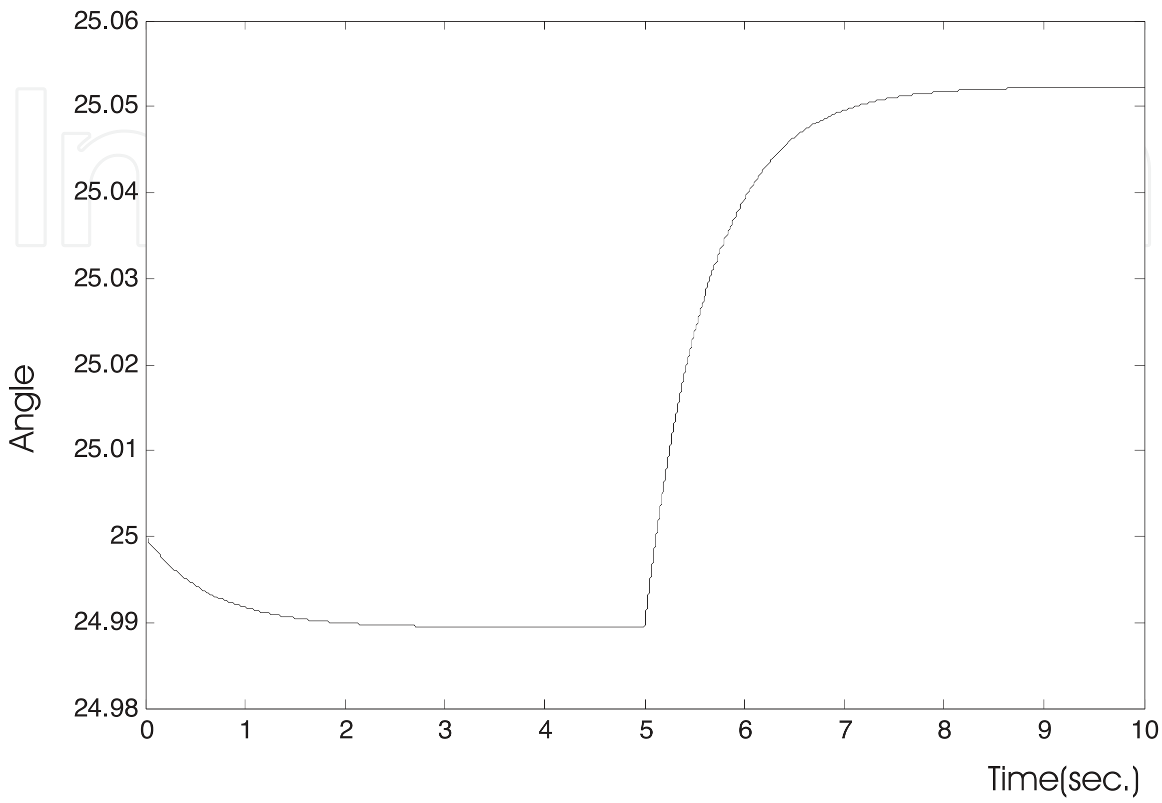

Fig. 7. Transient response of the robotic system with initial condition and reference input signal of 25°. An abrupt change is applied in moment of inertia from $J_2 = 25000$ to $J_2 = 50000$

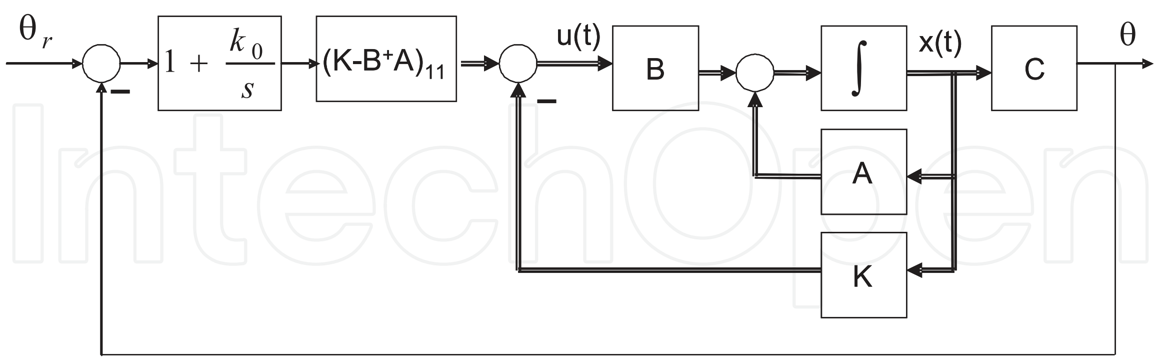

Fig. 8. A block diagram of the proposed controller with a PI controller to eliminate the steady state error

As can be seen in figure 7, the lack of precision in the model leads to a steady state error in the transient response. We propose a solution to eliminate this error. A simple but effective solution is realized by adding a feedback loop and including a PI controller as shown in figure 8.

$$e(t) = \theta_r(t) - \theta(t)$$

$$u = (K - B^{+}A)_{11}(e(t) + k_0 \int_{t_0}^{t} e(\tau)d\tau) + Kx(t)$$

Using the design shown in figure 8 and repeating the same experiment explained before with $k_0 = 1.5$ initial condition and reference input signal of 25°, keeping the moment of inertia constant with $J_2 = 25000$ and after five seconds an abrupt change is applied in the moment of inertia to $J_2 = 50000$. The result is shown in figure 9. It can be observed that a small disturbance effect is occurred in the output angle but it is immediately corrected resulting in a smooth response with zero steady state error. Figure 10 shows the response with an intial condition and reference input signal of 25°. The response is initiated with moment of inertia $J_2 = 25000$ and after five seconds an abrupt change is applied in the moment of inertia to $J_2 = 50000$. It can be easily noticed that the response has not been affected with the modification made to the propsed controller shown in figure 8 and the response is exactly similar to that shown in figure 6.

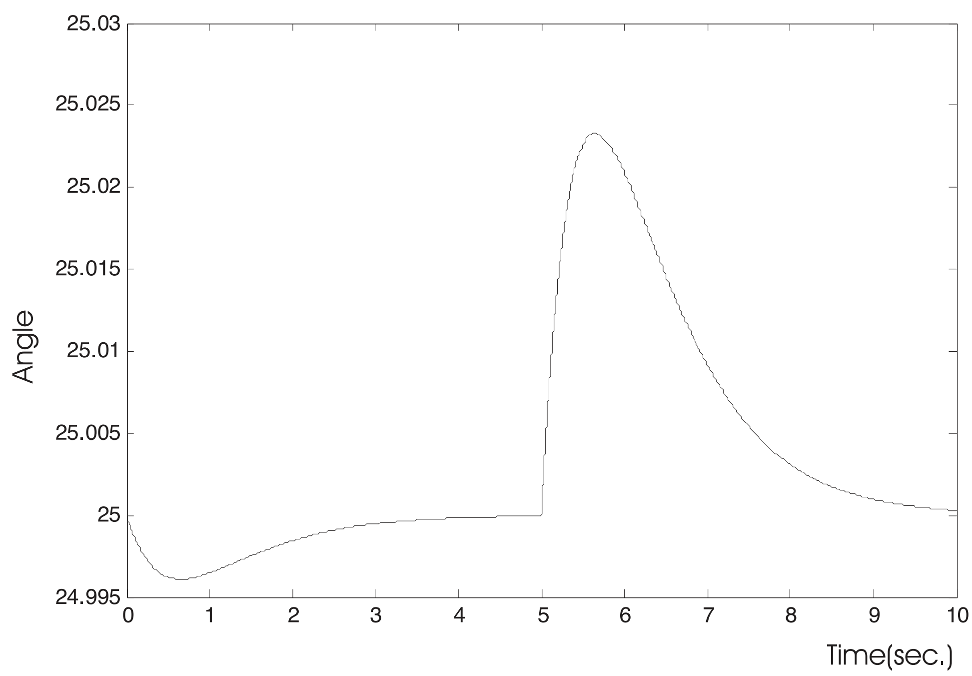

Fig. 9. Transient response of the robotic system by adding a PI controller to the proposed FC-LQR with initial condition and reference input signal of 25°. An abrupt change is applied in moment of inertia from $J_2 = 25000$ to $J_2 = 50000$

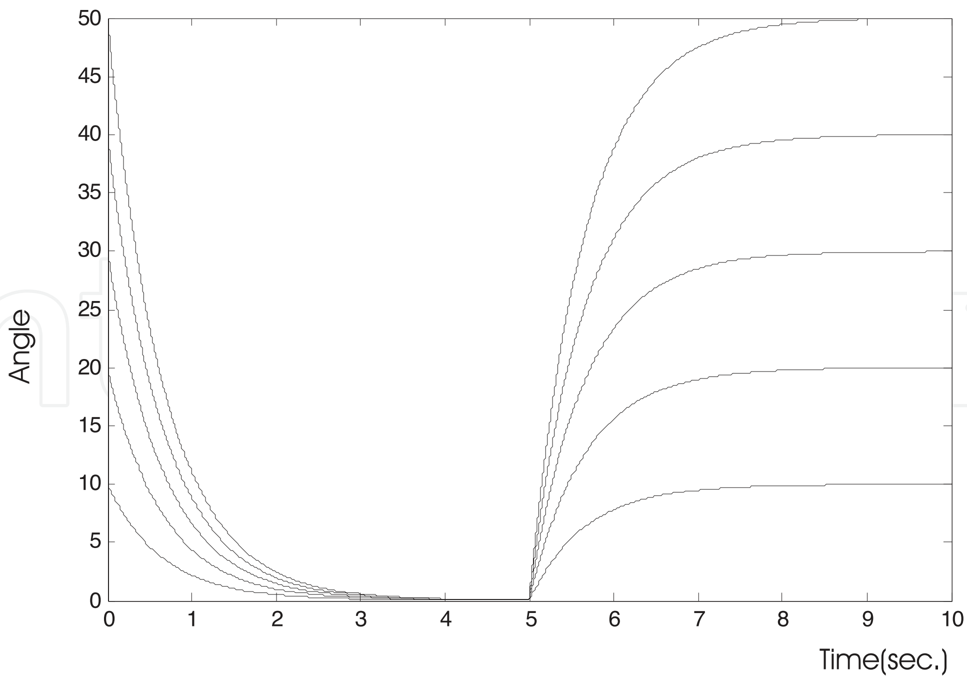

Fig. 10. Transient response of the robotic system by adding a PI controller to the proposed FC-LQR with various initial conditions and reference input signals and constant moment of inertia of $J_2 = 25000$

7. Conclusion

A robust FC-LQR for the control of a robotic system has been designed. The main idea is to design a supervisory fuzzy controller capable to adjust the controller parameters in order to obtain the desired axes positions under variations of the robot parameters and payload variations. The motivation behind this scheme is to combine the best features of fuzzy control and that of the optimal LQR.

Both the controlled system and the fuzzy controller are represented by the affine T-S fuzzy model taking into consideration the effect of the constant term. In the case of fuzzy control, the fuzzy system is resulted from blending all the sub-systems. The blending of the independent term of each rule will no longer be a constant but a function of the variables of the system and thus affects the dynamics of the resultant system. A necessary condition has been added to deal with the independent term.

In this chapter, we have demonstrated that the LQR, can be made more appropriate for actual implementation by introduction of fuzzy rules. The results obtained show a robust and stable behavior when the system is subjected to various initial conditions, moment of inertia and to disturbances.

8. References

[Berstecher et al. 2001] R.G. Berstecher, R. Palm, H.D. Unbehauen, An adaptive fuzzy sliding mode controller, IEEE Trans. Indust. Electron. 8 (1) (2001) 18Ű31.

[Burg et al. 2001] T. Burg, D. Dawson, M.D. Queiroz, An adaptive partial state feedback controller for RLED robot manipulators, IEEE Trans. Automat. Control 41 (7) (1996) 1024Ű1030.

[Canudas and Fixot 1991] C. Canudas de Wit, N. Fixot, Robot control via robust estimated state feedback, IEEE Trans. Automat. Control 36 (12) (1991) 1497Ű1501.

[Chen et al. 1998] B.S. Chen, H.J. Uang, C.S. Tseng, Robust tracking enhancement of robot systems including motor dynamics: a fuzzy-based dynamic game approach, IEEE Trans. Fuzzy Systems 6 (1998) 538Ű552.

[Chuan-Kai Lin 2003] Chuan-Kai Lin, A reinforcement learning adaptive fuzzy controller for robots, Fuzzy Sets and Systems 137 (3) (2003) 339Ű352.

[Commuri et al. 1996] S. Commuri, F.L. Lewis, Adaptive-fuzzy logic control of robots manipulators, Proc. IEEE Internat. Conf. on Robotics and Automation, Mineapolis, MI, 1996, pp. 2604Ű2609.

[Freund. 1982] Freund, E. Fast Nonlinear Control with Arbitrary Pole-placement for Industrial Robots and Manipulators, Robotics Research. pages 65-78 **1** (1982).

[Fuchun et al. 2003] Fuchun Sun, Zengqi Sun, Lei Li, Han-Xiong Li, Neuro-fuzzy adaptive control based on dynamic inversion for robotic manipulators, Fuzzy Sets and Systems 134 (1) (2003) 117Ű133.

[Fukuda et al. 1992] T. Fukuda, T. Shibata, Hierarchical intelligent control for robotic motion by using fuzzy, artificial intelligence, and neural networks, Proc. Internat. J. Conf. on Neural Networks, Vol. 1, Baltimore, 1992, pp. 269Ű274.

[Gamboa 1996] Gamboa, E., Control de robots con accionamiento hidráulicos. Aplicación a robots de grandes dimensiones para la construcción. Tesis doctoral. Universidad Politécnica de Madrid (1996).

[Hua et al. 2004] C. C. Hua, X. Guan, G. Duan, Variable structure adaptive fuzzy control for a class of nonlinear time delay systems, Fuzzy Sets and Systems 148 (2004) 453Ű468.

[Ishikawa 1988] Ishikawa, T. A study on fuzzy control of an arm robot. B. E. thesis, Dept. Contr. Eng., Tokyo Inst. Technol(1988).

[Jagannathan et al. 1996] S. Jagannathan, F.L. Lewis, Discrete-time adaptive fuzzy logic control of robotic systems, Proc. IEEE Internat. Conf. on Robotics and Automation, Mineapolis, MI, 1996, pp. 2586Ű2591.

[Khosla and Kanade 1988] P.K. Khosla, T. Kanade, Experimental evaluation of nonlinear feedback and feedforward control schemes for manipulator, Int. J. Robot. Res. 7 (1988) 18Ű28.

[Kim and Lewis 2000] Y.H. Kim, F.L. Lewis, Intelligent optimal control of robotic manipulators using neural networks, Automatica 36 (2000) 1355Ű1364.

[Kim and Lewis 1999] Y.H. Kim, F.L. Lewis, Neural network output feedback control of robot manipulators, IEEE Trans. Robot. Automat 15 (1999) 301Ű309.

[Koditschek. 1984] D. Koditschek, Natural motion for robot arms, Proc. 1984 IEEE Conf. on Decision and Control, Las Vegas, 1984, pp. 733Ű735.

[Labiod et al. 2005] S. Labiod, M.S. Boucherit, T.M. Guerra, Adaptive fuzzy control of a class of MIMO nonlinear systems, Fuzzy Sets and Systems 15 (1) (2005) 59Ű77.

[Lewis et al. 1994] F.L. Lewis, C.T. Abdallah, D.M. Dawson, Control of Robot Manipulators, MacMillan Publishing Company, New York, 1994.

[Li et al. 2001] W. Li, X.G. Chang, F.M. Wahl, J. Farrell, Tracking control of a manipulator under uncertainty by FUZZY P+ID controller, Fuzzy Sets and Systems 122 (1) (2001) 125Ű137.

[Llama et al. 2001] Miguel A. Llama, Rafael Kelly, Victor Santibañez. A stable motion control system for manipulators via fuzzy self-tuning. In *Fuzzy Sets ans Systems*, volume 124, pages 133 Ű154, 2001.

[Llama et al. 1998] M.A. Llama, V. Santibanez, R. Kelly, J. Flores, Stable fuzzy self-tuning computed-torque control of robot manipulators, Proc. IEEE Internat. Conf. on Robotics and Automation, Leuven, 1998, pp. 2369Ű2374. B 29 (1999) 371Ű388.

[Luh 1983] J.Y.S. Luh, Conventional controller design for Industrial robots-a tutorial, IEEE Trans. Systems Man Cybernet. 13 (1983) 298Ű316.

[Meslin et al. 1992] J.M. Meslin, J. Zhou, P. Coiffet, Fuzzy dynamic control of robot manipulators: a scheduling approach, Proc. IEEE Internat. Conf. on Systems Man and Cybernetics, Le Tourquet, 1993, pp. 69Ű73.

[Middleton and Goodwin 1988] R.H. Middleton, G.C. Goodwin, Adaptive computed torque control for rigid link manipulators, System Control Lett. 10 (1988) 9Ű16.

[Purwar et al. 2005] Purwar, S., Kar I. N., Jha A. N.: Adaptive control of robot manipulators using fuzzy logic systems under actuator constraints. In *Fuzzy Sets and Systems*, volume 152, pages 651 Ű664, 2005.

[Sciavicco et al. 1996] L. Sciavicco, B. Siciliano, Modeling and Control of Robot Manipulators, McGraw-Hill Co., New York, 1996.

[Slotine and Li 1988] J.J.E. Slotine, W. Li, Adaptive manipulator control: a case study, IEEE Trans. Automat. Control 33 (1988) 995Ű1003.

[Slotine et al. 1983] J.J.E. Slotine, S.S. Sastry, Tracking control of nonlinear systems using sliding surface with application to robot manipulator, Int. J. Control 38 (1983) 465Ű492.

[Song et al. 2006] Song, Z., Yi J., Zhao D., Li X. A computed torque controller for uncertain robotic manipulator systems: Fuzzy approach. In *Fuzzy Sets ans Systems*, volume 154, pages 208 Ű226, 2006.

[Spong and Vidyasagar 1989] M.W. Spong, M. Vidyasagar, Robot Dynamics and Control, Wiley, New York, 1989.

[Sugeno and Kang 1988] Structure identification of fuzzy model, *Fuzzy Sets Syst.*, vol. 28, pp. 15-33, 1988.

[Sugeno and Tanaka 1988] Successive identification of a fuzzy model and its applications to prediction of a complex system, *Fuzzy Sets and Systems* 42 (1991) 315-334, 1991.

[Sun et al. 1999] F. Sun, Z. Sun, G. Feng, An adaptive fuzzy controller based on sliding mode for robot manipulator, IEEE Trans. Systems Man Cybernet. 29 (1999) 661Ű667.

[Sylvia et al. 2003] Sylvia Kohn-Rich, Henryk Flashner, Robust fuzzy logic control of mechanical systems, Fuzzy Sets and Systems 133 (1) (2003) 77Ű108.

[Takagi and Sugeno 1985] Takagi, T., Sugeno M.: Fuzzy Identification of Systems and Its Applications to Modeling and Control. In *IEEE Transactions on Systems, Man and Cybernetics*, volume SMC-15, pages 116–132, 1985.

[Tanaka et al. 1996] K. Tanaka and T. Ikeda and H. O. Wang, Robust Stabilization of a Class of Uncertain Nonlinear Systems via Fuzzy Control: Quadratic Stabilizability, H_∞ Control Theory, and Linear Matrix Inequalities, IEEE Transactions on Fuzzy Systems, February, number 1, 1996, volume 4, pages 1-13.

[Tanaka and Sano 1994] K. Tanaka and M. Sano, A Robust Stabilization Problem of Fuzzy Control Systems and Its Application to Backing up Control of a Truck-Trailer, IEEE Transactions on Fuzzy Systems, May, number = "2", 1994, volume 2, pages 119-134.

[Tong et al. 2000] Tong Shaocheng, Tang Jiantao, Wang Tao, Fuzzy adaptive control of multivariable nonlinear systems, Fuzzy Sets and Systems 111 (2) (2000) 153Ű167.

[Tsai et al. 2000] C.H. Tsai, C.H.Wang, W.S. Lin, Robust fuzzy model following control of robot manipulators, IEEE Trans. Fuzzy Systems 8 (4) (2000) 462Ű469.

[Tzes et al. 1993] A. Tzes, K. Kyriakides, Adaptive fuzzy-control for Uexible-link manipulators: a hybrid frequency-time domain scheme, Proc. 2nd IEEE Internat. Conf. on Fuzzy Systems, San Francisco, 1993, pp. 122Ű127.
[Wang and mendel 1992] L.X. Wang, J.M. Mendel, Fuzzy basis function universal approximation, and orthogonal least square learning, IEEE Trans. Neural Networks 3 (1992) 807Ű814.
[Wijesoma and Richards 1990] S.W. Wijesoma, R.J. Richards, Robust trajectory following of robots using computed torque structure with VSS, Int. J. Control 52 (1990) 935Ű962.
[Yi and Chung 1997] S.Y. Yi, M.J. Chung, A robust fuzzy logic controller for robot manipulators with uncertainties, IEEE Trans. Systems Man Cybernet. Part B 27 (4) (1997) 706Ű713.
[Yoo and Ham 2000] B.K. Yoo, W.C. Ham, Adaptive control of robot manipulators using fuzzy compensator, IEEE Trans. Fuzzy Systems 8 (2) (2000) 186Ű199.
[Zhang et al. 2000] F. Zhang, D.M. Dawson, M.S. Queiroz de, W.E. Dixon, Global adaptive output feedback tracking control of robot manipulators, IEEE Trans. Automat. Control 45 (6) (2000) 1203Ű1208.
[Zhou et al. 1992] J. Zhou, P. Coi8et, Fuzzy control of robots, Proc. lst Internat. Conf. on Fuzzy Systems, San Diego, 1992, pp. 1357Ű1364.

Robot Manipulators New Achievements

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