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Optical and Infrared Modeling

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1. Introduction

In order to understand the relationships between the vegetation features (namely amount and structure) and the amount of sunlight reflected in the visible and near- to middle-infrared spectral domains many empirical methods based on various vegetation indices (e.g., NDVI, EVI) (Kallel et al., 2007), and physical approach namely based on radiative transfer (RT) theory have been developed. In RT, two model types can be distinguished: (i) one-Dimensional (1-D) models providing a (semi)analytical expression of the Bidirectional Reflectance Distribution Function (BRDF) of canopy architecture, its scattering parameters, and scene geometry (Gobron et al., 1997; Verhoef, 1984; 1998); (ii) 3-D models based on Monte Carlo simulations of a large number of photons randomly propagating through a canopy (Gastellu-Etchegorry et al., 1996; Lewis, 1999; North, 1996). Compared to 1-D models, such 3-D methods allow to take into account canopy heterogeneity with high accuracy. However, they suffer from long running times making their inversion difficult.

The RT theory was first proposed by Chandrasekhar (1950) to study radiation scattering in conventional (i.e. rotationally invariant) media. Such an assumption could be sufficient to model, for example, light scattering in the atmosphere, but appears rudimentary for modeling the reflectance of leaves, or shoots, in a vegetation canopy. To extend the formulation to such a case, many models are proposed. Among the 1-D model, one can cite SAIL (Verhoef, 1984) that is among the most widely used in case of turbid (null size components) crops canopies. The SAIL model allows to derive a non-isotropic BRDF considering two diffuse fluxes (upward/downward flux) to model the multiple scattering of the radiant flux by the vegetation elements. These fluxes are assumed to be semi-isotropic, which is only an approximation that lead to reflectance underestimation (Pinty et al., 2004). As a solution, Verhoef (1998) developed SAIL++ which is a 1-D model providing accurate reflectance estimation in the turbid case. Indeed, this model divides the diffuses fluxes into 72-subfluxes, and turns the SAIL equation system into a matrix-vector equation. Compared to 3-D models of RAMI 2 database in the turbid case (Pinty et al., 2004), SAIL++ gives accurate results.

Another solution to overcome the semi-isotropy assumption in the turbid case will be presented in this chapter, it is based on the coupling between SAIL and Adding method (Cooper et al., 1982; Van de Hulst, 1980). For such a method, optical characteristics of canopy layers such that reflectance and transmittance are directly defined and handled at the scale of the vegetation layer (as operators). Their physical interpretation is hence easier. However, the vegetation description is rather simplistic and the canopy internal geometry is represented with low accuracy. Indeed, in order to retrieve the adding operators for each layer, Cooper et al. (1982) did not take into account the high order interactions between light and vegetation

which are very important as shown in (Pinty et al., 2004). In order to adapt the Adding method to such a configuration, we need a more accurate estimation of the Adding scattering parameters. Since the Adding method operators are derived from the bidirectional reflectance and transmittance of the considered layer, in this study we propose to introduce the SAIL canopy description into the Adding formulation. The developed model is called AddingS.

Now, since the size of vegetation elements cannot be assumed null. Among others, Kuusk (1985) proposed a correction allowing the extension of the RT models like SAIL and SAIL++ to the discrete case (non-null-size components) (Verhoef, 1998). This approach allows to take into account the hot spot effect representing the bright area in the direction opposite to the direction of a pointlike the light source. This effect is caused by the high probability of backscattering which is proportional to the mean size of medium elements. Such an approach suffers from a severe shortcoming: compared to the turbid case, it increases only the reflectance created by the first collision of the radiation by leaves. As this increasing is not followed by the decreasing of other fluxes, it leads to a violation of the energy conservation law (Kallel, 2007). Therefore, based on the Kuusk (1985) approach, we propose the adaptation of AddingS to the discrete case. The extended model is called AddingSD. This model allows both to conserve the energy and to take into account the hot spot effect between diffuse fluxes. As AddingS/AddingSD are based on adding method then they need a long running time for that in this study, we benefit from both the rapidity of the SAIL++ as well as the hot spot modeling in the AddingSD and we propose a new other approach. This approach is based on the tracking of the flux created by the first photons collision by leaves. The analysis of this flux will be done using AddingSD and the RT problem resolution will be based on SAIL++.

The chapter is divided up as follows. First, we present the theoretical background of our models (Section 2). Then, we show model implementation (Section 3), and some validation results (Section 4). Finally, we present our main conclusions and perspectives (Section 5).

2. Theoretical background

In this section, we will first present the models AddingS/AddingSD then we expose our model based on flux decomposition.

2.1 AddingS/AddinSD modeling

The Adding method is based on the assumption that a vegetation layer receiving a radiation flux from bottom or top, partially absorbs it and partially scatters it upward or downward, independently of the other layers (Cooper et al., 1982; Van de Hulst, 1980). Thus, the relationships between fluxes are given by operators which allow the calculation of the output flux density distribution as a function of the input flux density distribution. As the Adding method vegetation layer operators depend on the bidirectional reflectance and transmittance, we propose to derive them both in the turbid and the discrete case based on respectively SAIL and the Kuusk definition of the Hot Spot.

In this section, we first present the Adding operator definition, and secondly the derivation of the bidirectional reflectance and transmittance of a vegetation layer in both turbid and discrete cases corresponding respectively to the operators of the models AddingS and AddingSD.

2.1.1 Adding operators reformulation in the continuous case

In this paragraph, we present a generalization of the Adding operators presented in (Cooper et al., 1982) in the continuous case, dealing with radiance hemispherical distribution.

For a given medium having two parallel sides (top and bottom) receiving a source radiation flux $dE_i(\Omega_i = (\theta_i, \varphi_i))$ (θ_i the zenithal angle and φ_i the azimuthal angle) provided within a cone of solid angle $d\Omega_i = \sin(\theta_i)d\theta_id\varphi_i$, produces elementary radiances at the top and the bottom of the medium called respectively $dL_e(\Omega_i, \Omega_e)$ and $dL'_e(\Omega_i, \Omega'_e)$ in the directions $\Omega_e = (\theta_e, \varphi_e)$ and $\Omega'_e = (\theta'_e, \varphi'_e)$, respectively.

So the BRDF, r , and the bidirectional transmittance distribution function (BTDF), t , are defined respectively as follows:

$$\begin{aligned} r(\Omega_i \rightarrow \Omega_e) &= \frac{\pi dL_e(\Omega_i, \Omega_e)}{dE_i(\Omega_i)} = \frac{\pi dL_e(\Omega_i, \Omega_e)}{L_i(\Omega_i) \cos(\theta_i) d\Omega_i}, \\ t(\Omega_i \rightarrow \Omega'_e) &= \frac{\pi dL'_e(\Omega_i, \Omega'_e)}{dE_i(\Omega_i)} = \frac{\pi dL'_e(\Omega_i, \Omega'_e)}{L_i(\Omega_i) \cos(\theta_i) d\Omega_i}. \end{aligned} \quad (1)$$

where L_i is the radiance provided by the source.

So, we define the two scattering operators $\mathcal{R}$ and $\mathcal{T}$, that give the outward radiance L_e from an incident radiance defined over the whole hemisphere L_i :

$$\mathcal{R}[L_i](\cdot) = \frac{1}{\pi} \overbrace{\int_{\Pi}}^{\text{Over hemisphere}} r(\Omega_i \rightarrow \cdot) L_i(\Omega_i) \cos(\theta_i) d\Omega_i, \quad (2)$$

$$\mathcal{T}[L_i](\cdot) = \frac{1}{\pi} \int_{\Pi} t(\Omega_i \rightarrow \cdot) L_i(\Omega_i) \cos(\theta_i) d\Omega_i. \quad (3)$$

For two medium 1 and 2 such that the second one is above the first one, the top reflectance operator for the canopy is given by (Verhoef, 1985):

$$\mathcal{R}_t = \mathcal{R}_{t,2} + \mathcal{T}_{u,2} \circ (I - \mathcal{R}_{t,1} \circ \mathcal{R}_{b,2})^{-1} \circ \mathcal{R}_{t,1} \circ \mathcal{T}_{d,2}. \quad (4)$$

where $\mathcal{T}_{u,2}$, $\mathcal{T}_{d,2}$ are respectively the upward and downward transmittances of the layer 2, $\mathcal{R}_{t,1}$ and $\mathcal{R}_{b,1}$ are the reflectances of respectively the top of layer 1 and the bottom of layer 2, and I is the identity operator.

Finally, to be implemented such operators have to discretized. Thus, Kallel et al. (2008) propose a regular discretization of the zenithal angle θ and azimuthal angle φ into 20 and 10 intervals respectively. In this case, the reflectance and transmittance operators become matrices and the 'o' operator becomes matrix multiplication.

2.1.2 Turbid case: AddingS

For one vegetation layer, the top and bottom reflectance operators and the downward and upward transmittance operators require the estimation of top and bottom bidirectional reflectances, the downward and upward bidirectional transmittance respectively, r_t , r_b , t_d and t_u . Now, assuming that the vegetation layer is formed by small and flat leaves with uniform azimuthal distribution, the layer has the same response when observed from the top or the bottom. $r_b = r_t$ and $t_u = t_d$. Moreover, two kinds of transmittances can be distinguished: those provided from the extinction of the incident flux, and those provided by the scattering of the incident flux by the vegetation components. So, we called them respectively $t_{.,s}$ and $t_{.,d}$, where . equals d (downward) or u (upward).

The SAIL model allows the BRDF (r_t) and the BTDF by scattering ($t_{d,d}$) derivation of a vegetation layer. Moreover, Kallel et al. (2008) showed that

$$t_{d,s}(\Omega_i \rightarrow \Omega'_e) = \frac{\tau_{ss} \delta(\theta'_e = \theta_i) \delta(\varphi'_e = \varphi_i)}{\cos(\theta_i) \sin(\theta_i)}, \quad (5)$$

with τ_{ss} the direct transmittance given by SAIL.

As such a model is based on SAIL which assumes that the diffuse fluxes are semi-isotropic, then it is only correct for thin layers ($LAI < 10^{-2}$) where the diffuse fluxes contribution to the BRDF/BTDF are small. Therefore, to estimate the reflectance of a thick layer and overcome the semi-isotropy assumption, we propose to divide the thick layer into thin sublayers with LAI value, $L_{min} = 10^{-3}$. The whole layer reflectance operator is then derived with good accuracy using the adding method Eq. (4) as it allows to model the diffuse flux anisotropy.

2.1.3 Discrete case: AddingSD

In the discrete case, the size of the leaves is no longer assumed null and there is a non-negligible correlation between the incident flux path and the diffused flux: the hot spot effect Kuusk (1985); Suits (1972). Until now, such an effect was taken into account in 1-D model only for the single scattering contribution from soil and foliage that is increased. Now, as the diffuse fluxes are not decreased consequently, the radiative budget is not checked. Now, the hot spot effect occurs also for diffuse fluxes (whose contribution increases with the vegetation depth). We call such a phenomena the multi hot spot effect. In this section, having recall Kuusk' model Kuusk (1985), we present our approach.

2.1.3.1 Kuusk' model

For a layer located at in altitude between -1 and 0, the single scattering reflectance ($\rho_{HS}^{(1)}$) by a leaf M at depth z , for the source and observation directions being respectively Ω_s and Ω_o , is (Verhoef (1998), pp 150-159):

$$\rho_{HS}^{(1)}(z) = P_{so}(\Omega_s, \Omega_o, z) \frac{w(\Omega_s, \Omega_o)}{\pi}, \quad (6)$$

where w is the bidirectional scattering parameter under the vegetation (Verhoef, 1984) and $P_{so}(\Omega_s, \Omega_o, z)$ is the conjoint probability that the incident flux reaches M without any collision with other canopy components and that, after scattering by M , it also reaches the top of the canopy without collisions Kuusk (1985):

$$\begin{aligned} P_o(\Omega_s, \Omega_o, z) &= \exp \left[- \int_z^0 \{k + K - \sqrt{kK} \exp[(z-x)b]\} dx \right], \\ &= \exp[(K+k)z] C_{HS}(\Omega_s, \Omega_o, z), \end{aligned} \quad (7)$$

with k, K the extinction respectively in source and observation directions and C_{HS} the correction factor:

$$C_{HS}(\Omega_s, \Omega_o, z) = \exp \left[\frac{\sqrt{kK}}{b} [1 - \exp(bz)] \right], \quad (8)$$

where b is a function of the vegetation features, the different solid angles and the hot spot factor d_l defined as the ratio between the leaf radius and the layer height Kuusk (1985); Pinty et al. (2004).

2.1.3.2 Multi hot spot model

Firstly recall that the energy conservation is insured by adding model whatever be the foliage area volume density (FAVD), u_l (cf. Appendix B) or the probability of finding foliage P_χ . In this subsection, we first show that the first order hot spot corresponds to the use of a fictive equivalent P_χ , called $P_{\chi,HS}$.

For a vegetation layer composed of two layers: a thin layer 2 above a layer 1, located respectively in $[z_0, 0]$ and $[-1, z_0]$, let $P_{so}(\Omega_s, \Omega_o, z_0, z)$ denotes the joint probability that the two fluxes do not collide with leaves for $z' \in [z_0, 0]$ (only in the layer 2). Its expression is obtained from Eq. (7) by changing the integral endpoints $[z, 0]$ by $[z_0, 0]$:

$$P_{so}(\Omega_s, \Omega_o, z_0, z) = \exp[(K + k)z_0] \mathcal{C}_{HS}(\Omega_s, \Omega_o, z_0, z),$$

with $\mathcal{C}_{HS}$ the generalized correction factor:

$$\mathcal{C}_{HS}(\Omega_s, \Omega_o, z_0, z) = \exp \left[\frac{\sqrt{kK}}{b} \left(\exp[b(z - z_0)] - \exp[bz] \right) \right].$$

The conditional probability definition that the flux in the direction Ω_o does not collide leaves given the same property for the incident flux is:

$$P_o(\Omega_o | \Omega_s, z_0, z) = \frac{P_{so}(\Omega_s, \Omega_o, z_0, z)}{P_s(\Omega_s, z_0)},$$

where $P_s(\Omega_s, z_0)$ represents the prior probability of gap in the direction Ω_s . Since $P_s(\Omega_s, z_0) = \exp[kz_0]$, then:

$$P_o(\Omega_o | \Omega_s, z_0, z) = \exp[Kz_0] \mathcal{C}_{HS}(\Omega_s, \Omega_o, z_0, z).$$

In the case of the direct flux, the first order contribution of a leaf $M(z)$ in the layer 1 to the BRDF is:

$$\rho_{HS}^{(1)}(z) = \underbrace{\exp[kz_0]}_{P_s(\Omega_s, z_0)} \underbrace{\rho_{HS}^{(1)}(z - z_0)}_{\text{layer 1}} \underbrace{\exp\left\{Kz_0 + \log[\mathcal{C}_{HS}(\Omega_s, \Omega_o, z_0, z)]\right\}}_{P_o(\Omega_o | \Omega_s, z_0, z)}. \quad (9)$$

In Eq. (9), $\rho_{HS}^{(1)}(z)$ can be interpreted as follows: reaching the top of the canopy the direct flux is partially extinguished in the layer 2 by the factor $P_s(\Omega_s, z_0)$. Then, reaching the interface between the two layers, its amplitude will be determined according to $\rho_{HS}^{(1)}(z - z_0)$ that depends on the layer 1 features. Finally, $K_{HS}(\Omega_o | \Omega_s, z_0, z)$ can be viewed as the 'effective' extinction related to the conditional probability of gap $P_o(\Omega_o | \Omega_s, z_0, z)$ of the layer 2. Indeed, $K_{HS} < K$ means that the probability of collision with leaves (or probability of finding leaves, P_χ) for the exiting flux that it will be noted $L_{o,HS}^{(1)}$ is decreased. Since the extinction depends linearly on P_χ , one can deem that P_χ is locally decreased by the factor $\gamma = \frac{K_{HS}}{K}$:

$$P_{\chi,HS}(\Omega_o | \Omega_s, z_0, z) = \frac{K_{HS}(\Omega_o | \Omega_s, z_0, z)}{K} P_\chi. \quad (10)$$

The physical interpretation of $P_{\chi,HS}$ is as follows. Assume that the probability of gap (for a given flux) is increased in the layer 2. For this flux, the 'effective' probability of being collided by vegetation when crossing the layer is reduced accordingly. Obviously, the first collision between the flux and the vegetation is reduced according to the same probability of finding vegetation or similarly the same density of vegetation. Now, since the layer 2 is thin, its corresponding reflectance and diffuse transmittance depend mainly on the first interaction. So, just an approximation of the multiple scattered fluxes is sufficient to derive the layer 2 scattering terms with good accuracy. For that, the derivation of all diffuse fluxes can be done using this 'effective' probability of finding foliage ($P_{\chi,HS}$ in our case). Moreover, for such a modeling, the

interactions of the considered flux and the layer 2 components (transmittance by extinction, reflectance and diffuse transmittance) are derived using exactly the same probability value ($P_{\chi,HS}$), which is physically consistent and thus leads to the conservation of the energy of this flux. Furthermore, by doing the same processing for all fluxes exiting the layer 1 in direction of the layer 2, the energy of all fluxes is conserved and so the energy is conserved in the system composed by the two vegetation layers.

The layer 2 reflectance and diffuse transmittance of the flux $L_{o,HS}^{(1)}$, respectively called $r_{b,2,HS}(z, \Omega_o \rightarrow .)$ and $t_{d,2,HS}(z, \Omega_o \rightarrow .)$, have therefore to be estimated using $P_{\chi,HS}$ rather than the initial P_{χ} . The first order hot spot effect can then be viewed as a local reduction of the layer 2 probability of finding leaves. The layer 2 operators are derived accordingly, and the two layer reflectance operator is obtained using Eq. (4). In summary, given a vegetation layer, its corresponding reflectance is computed dividing it into N_{HS} thin sublayers with a value of LAI, $L_{HS} = 3 \times 10^{-2}$ (L_{HS} is higher than the elementary sublayer LAI corresponding to AddingS model concatenation, $L_{min} = 10^{-3}$) and iteratively adding a new sublayer to the current 'stack' of sublayers (from 1 to N_{HS}).

More precisely, beginning from a thin layer, where the neglecting of the hot spot effect appears reasonable, thin layers are added, one after one, to build up a 'system' taking into account the whole hot spot effect (as well as conserving the energy). The contribution of each new sublayer 2 to the high order hot spot effect is computed as follows. The flux reaching the top of the layer 2 is scattered many times before reaching the interface between the two layers where it is considered again as a direct flux (according to the adding method). In layer 1, the first order (direct flux case) hot spot computation is therefore valid. Adding iteratively the thin layers and the contribution of their diffuse fluxes, the hot spot effect between all the diffuse fluxes is taken into account.

Finally, for more information about the implementation of the models AddingS/AddingSD, readers are invited to read the article (Kallel et al., 2008).

2.2 Virtual flux decomposition

In this section, we propose an alternative to AddingSD that is simpler, conserves the energy and based on effective vegetation density too but does not take into account the high order hot spot effect. Moreover, the proposed approach is an extension to the discrete case of SAIL++, that we provide an overview in Appendix A. To do the extension, we study the collision of direct fluxes with vegetation in the discrete homogeneous medium case. The energy will be conserved by increasing the flux created by first collision and decreasing the flux created by this flux scattering.

2.2.1 Derivation of $L_+^{1,n}$

Figure 1 shows two points $M(x, y, z)$ and $N(x', y', t)$ in a vegetation layer assumed be a homogeneous discrete medium such that $t < z$. The elementary volume at M is viewed from N at an elementary solid angle $d\Omega$ with $\Omega = (\theta, \varphi)$. A direct flux ($E_s(0)$) present above the vegetation layer having direction $\Omega_s = (\theta_s, 0)$ passes through the vegetation from the top to N without a collision. By assuming a constant extinction k along the path, E_s at altitude t is

$$E_s(t) = E_s(0) \exp(kt). \quad (11)$$

Then the light is scattered in an elementary volume at N with an elementary thickness dt . Thus scattered radiance in the direction $d\Omega$ called ($dL_+^1(N, \Omega)$) is

$$dL_+^1(N, \Omega) = E_s(t) \pi^{-1} w(\Omega_s \rightarrow \Omega) dt. \quad (12)$$

Fig. 1. A vegetation layer located from altitude 0 to $-H$ and assumed a discrete medium. Two point $M(x, y, z)$ and $N(x', y', z)$ are located in the layer. The elementary volume at M is viewed from N under an elementary solid angle ($d\Omega$) with a polar angle θ . A direct flux with zenith angle θ_s collides with vegetation in an elementary volume at point N , is then reflected in the solid angle $d\Omega$ and reaches point M without collision. The downward and upward paths are correlated from altitude z to t as shown by the gray triangle linking the two paths.

$dL_+^1(N, \Omega)$ travels from N to M without collision. Therefore, by assuming a constant extinction κ along the path and without taking into account the dependency between paths, the radiance reaching M called $dL_+^{1*}(N \rightarrow M, \Omega)$ is

$$\begin{aligned} dL_+^{1*}(N \rightarrow M, \Omega) &= dL_+^1(N, \Omega) \exp[\kappa(t - z)], \\ &= E_s(0) \exp[(k + \kappa)(t - z)] \exp(kz) \pi^{-1} w(\Omega_s \rightarrow \Omega) dt. \end{aligned} \quad (13)$$

Since the medium is assumed discrete, the hot spot effect representing the dependency between downward direct fluxes and diffuse fluxes at N has to be taken into account from depth t to z . Using Kuusk's model [1985], the radiance reaching M called $dL_+^1(N \rightarrow M, \Omega)$ is

$$\begin{aligned} dL_+^1(N \rightarrow M, \Omega) &= dL_+^{1*}(N \rightarrow M, \Omega) \exp \left[\frac{\sqrt{k\kappa}}{b} (1 - \exp[-b(z - t)]) \right], \\ &= E_s(0) \exp[(k + \kappa)(t - z)] \exp \left[\frac{\sqrt{k\kappa}}{b} (1 - \exp[-b(z - t)]) \right] \\ &\quad \times \exp(kz) \pi^{-1} w(\Omega_s \rightarrow \Omega) dt. \end{aligned} \quad (14)$$

Eq. (14) is the foundation of our model. However, since it has a complex expression, in particular in the exponential term corresponding to the hot spot correction, there is no linearity versus z and t enabling a simple solution based on differential equations as those of SAIL++ [cf. Eqs. 85]. For that, we propose to apply the Taylor series decomposition to this term

$$\exp \left[\frac{\sqrt{k\kappa}}{b} (1 - \exp[-b(z - t)]) \right] = \exp \left[\frac{\sqrt{k\kappa}}{b} \right] \sum_{n=0}^{\infty} \frac{(-1)^n (k\kappa)^{n/2}}{n! b^n} \exp[nb(t - z)]. \quad (15)$$

Therefore, Eq. (14) can be written as follows,

$$dL_+^1(N \rightarrow M, \Omega) = \sum_{n=0}^{\infty} (-1)^n A_n dL_+^{1,n}(N \rightarrow M, \Omega), \quad (16)$$

where

$$\begin{aligned} dL_+^{1,n}(N \rightarrow M, \Omega) &= E_s(0) \exp[(k + \kappa_n)(t - z)] \times \exp(kz) \pi^{-1} w(\Omega_s \rightarrow \Omega) dt, \\ A_n &= \frac{(k\kappa)^{n/2}}{n!b^n} \exp\left[\frac{\sqrt{k\kappa}}{b}\right], \\ \kappa_n &= \kappa + nb. \end{aligned} \quad (17)$$

As the vegetation is homogeneous, then $dL_+^1(N \rightarrow M, \Omega)$ can be written simply as $dL_+^1(t \rightarrow z, \Omega)$. Thus, $L_+^1(z, \Omega)$ is obtained by integration of dL_+^1 over the depth $[-H, z]$

$$L_+^1(z, \Omega) = \int_{t=-H}^z dL_+^1(t \rightarrow z, \Omega). \quad (18)$$

Based on (16), L_+^1 can be written as

$$L_+^1(z, \Omega) = \sum_{n=0}^{+\infty} (-1)^n A_n L_+^{1,n}(z, \Omega), \quad (19)$$

where

$$\begin{aligned} L_+^{1,n}(z, \Omega) &= \int_{-H}^z E_s(0) \exp[(k + \kappa + nb)(t - z)] \exp(kz) \pi^{-1} w(\Omega_s \rightarrow \Omega) dt, \\ &= E_s(0) \frac{1 - \exp[-(k + \kappa + nb)(H + z)]}{k + \kappa + nb} \exp(kz) \pi^{-1} w(\Omega_s \rightarrow \Omega). \end{aligned} \quad (20)$$

2.2.2 Application of the effective vegetation density approach

Here, we will try to extend the reformulated SAIL++ equation (cf. Appendix A.3) to the discrete case. Thus, as shown in Section 2.1.3, the hot spot effect will be treated as an increased posterior probability of gap which, in turn, results from a reduction in vegetation density. Then, it was suggested the use of the concept 'effective vegetation density' to describe the phenomenon. In this subsection, we propose to derive this density for $L_+^{1,n}, \forall n \in \mathbb{N}$, and to use it further to derive the equations of fluxes created by $L_+^{1,n}$ scattering. Moreover, the same effective density using leads to conserve energy (as explained in Section 2.1.3).

In Eqs. (13) (17), the difference between dL_+^{1*} and $dL_+^{1,n}$ is the value of the extinction in the direction Ω (κ and κ_n respectively). Note that $\forall n > 0, \kappa_n > \kappa$, then $dL_+^{1,n}$ decreases faster than dL_+^1 .

According to our approach described in Section A, the variation in the extinction factor is linked to the variation of the collision probability locally around M . In other words, a decrease in the probability of finding foliage at M decreases P_χ , accordingly (cf. Appendix B). Now, according to (77) and (99)

$$\left. \begin{aligned} \kappa &= d_L P_\chi(M) \kappa_o \\ \kappa_n &= d_L P_{\chi,n}(M) \kappa_o \end{aligned} \right\} \Rightarrow P_{\chi,n}(M) = \frac{\kappa_n}{\kappa} P_\chi(M), \quad (21)$$

with $P_{\chi,n}(M)$ the a posteriori probability of finding vegetation at M for the virtual radiance $dL_+^{1,n}$, and κ_0 the normalized extinction factor [as explained in Eq. (77), it is independent on vegetation density]. We will use this notation in the following for SAIL++ scattering parameters. For each scattering parameter X , one can define the corresponding normalized one X_0 according to Eq. (77).

As we can see in Eqs. (21), $P_{\chi,n}(M)$ does not depend on M . Thus, it will be simply called $P_{\chi,n}$. Then, based on L_+^1 differential equation derivation [cf. Eq. (89)] and replacing κ by κ_n , we obtain,

$$\begin{aligned} \frac{dL_+^{1,n}(z, \Omega)}{dz} &= [\mathfrak{s} \circ E_s(z, \Omega_s)](\Omega) - \kappa_n L_+^{1,n}(z, \Omega) = [\mathfrak{s} \circ E_s(z, \Omega_s)](\Omega) - d_L P_{\chi,n} \kappa_0 L_+^{1,n}(z, \Omega), \\ &= [\mathfrak{s} \circ E_s(z, \Omega_s)](\Omega) - d_L P_{\chi,n} [\mathfrak{k}_0 \circ L_+^{1,n}(z)](\Omega), \end{aligned} \quad (22)$$

where $\mathfrak{k}_0$ is the normalized scattering term corresponding to $\mathfrak{k}$ [cf. Eq. (80)].

It leads to the following important result linking the differentiation of L_+^1 to $(L_+^{1,n})_{n \in \mathbb{N}}$:

$$\begin{aligned} \frac{dL_+^1(z, \Omega)}{dz} &= \frac{d \left\{ \sum_{n=0}^{+\infty} (-1)^n A_n L_+^{1,n}(z, \Omega) \right\}}{dz}, \\ &= \sum_{n=0}^{+\infty} (-1)^n A_n \{ [\mathfrak{s} \circ E_s(z, \Omega_s)](\Omega) - d_L P_{\chi,n} [\mathfrak{k}_0 \circ L_+^{1,n}(z)](\Omega) \}, \\ &= [\mathfrak{s} \circ E_s(z, \Omega_s)](\Omega) \underbrace{\sum_{n=0}^{+\infty} (-1)^n A_n}_{=1} - d_L \sum_{n=0}^{+\infty} (-1)^n A_n P_{\chi,n} [\mathfrak{k}_0 \circ L_+^{1,n}(z)](\Omega), \\ &= [\mathfrak{s} \circ E_s(z, \Omega_s)](\Omega) - d_L \sum_{n=0}^{+\infty} (-1)^n A_n P_{\chi,n} [\mathfrak{k}_0 \circ L_+^{1,n}(z)](\Omega). \end{aligned} \quad (23)$$

Thus, the radiance distributions created by $dL_+^{1,n}$ scattering depend on $P_{\chi,n}$ rather than P_χ . As explained in Appendix A.3, these radiances are the downward diffuse radiance distribution (L_-), upward higher order diffuse radiance distribution (L_+^∞), upward radiance in observation direction (E_o^+) and downward radiance in observation direction (E_o^-). Note that, the mathematical validation, in term of global flux estimation, is explained in Subsection 2.2.3 and then shown in Appendix C.

Note that, similar to L_+^1 , the differentiation of E_o^+ that depends only on E_s is

$$\frac{dE_o^+(z, \Omega_o)}{dz} = w E_s(z, \Omega_s) - d_L \sum_{n=0}^{+\infty} (-1)^n A_n P_{\chi,n} K_0 E_o^{+,n}(z, \Omega_o), \quad (24)$$

with Ω_o the E_s direction, K the extinction factor in the direction Ω_o and

$$E_o^{+,n}(z, \Omega_o) = E_s(0) \frac{1 - \exp[-(k + K + nb)(H + z)]}{k + K + nb} \exp(kz) w(\Omega_s \rightarrow \Omega_o). \quad (25)$$

As in classical models, there is no need to use Eq. (24). We merely assume, as in the turbid case, that

$$\frac{dE_o^+(z, \Omega_o)}{dz} = w E_s(z, \Omega_s) - K E_o^+(z, \Omega_o), \quad (26)$$

and the reflectance provided from the first order collision ($\rho_{so}^{(0),HS}$) will be corrected using the traditional formula (Kuusk, 1985)

$$\rho_{so}^{(0),HS} = w \int_{-H}^0 \exp \left[(k + K)z + \frac{\sqrt{kK}}{b} [1 - \exp(bz)] \right] dz. \quad (27)$$

2.2.3 Dependency on $L_+^{1,n}$

In this subsection, we propose a modification to the reformulated SAIL++ equation set, presented in Appendix A.3, in order to take into account the effective vegetation density values in the expressions of L_- , L_+^∞ , E_o^+ and E_o^- that depend on $L_+^{1,n}$ scattering.

First, let us derive the angular differentiation of E_o^+ ($d^2 E_o^+(z, \Omega \rightarrow \Omega_o)$) that depends only on $L_+^{1,n}$. Compared to the dependency on L_+ in classical SAIL++ equations, P_χ has to be replaced by $P_{\chi,n}$. Thus,

$$\frac{d[d^2 E_o^+(z, \Omega \rightarrow \Omega_o)]}{dz} = w'_n(\Omega \rightarrow \Omega_o) L_+^{1,n}(z, \Omega) \cos(\theta) d\Omega, \quad (28)$$

where

$$w'_n(\Omega \rightarrow \Omega_o) = d_L P_{\chi,n} w'_0(\Omega \rightarrow \Omega_o), \quad (29)$$

with w'_0 the normalized scattering parameter corresponding to w' [cf. Eq. (76)].

Then, the angular differentiation of E_o^+ ($d^2 E_o^+(z, \Omega \rightarrow \Omega_o)$) which depends only on $L_+^{1,n}$ is obtained by summing the contribution of the set $(L_+^{1,n})_{n \in \mathbb{N}}$

$$\frac{d[d^2 E_o^+(z, \Omega \rightarrow \Omega_o)]}{dz} = \sum_{n=0}^{+\infty} (-1)^n A_n w'_n(\Omega \rightarrow \Omega_o) L_+^{1,n}(z, \Omega) \cos(\theta) d\Omega. \quad (30)$$

Note that, based on AddingSD formalism, the validity of our decomposition in this derivation of $P_{\chi,n}$ is shown in Appendix C.

By integration of Ω over the upper-hemisphere [cf. Eqs. (75) (84)], Eq. (30) becomes

$$\begin{aligned} \frac{dE_o^+(z, \Omega_o)}{dz} &= d_L \sum_{n=0}^{+\infty} (-1)^n A_n P_{\chi,n} \int_{\Pi} w'_0(\Omega \rightarrow \Omega_o) L_+^{1,n}(z, \Omega) \cos(\theta) d\Omega, \\ &= d_L \sum_{n=0}^{+\infty} (-1)^n A_n P_{\chi,n} [\mathbf{v}'_0 \circ L_+^{1,n}(z)], \end{aligned} \quad (31)$$

with $\mathbf{v}'_0$ the normalized scattering parameter corresponding to $\mathbf{v}'$ [cf. Eq. (74)].

Next, by integrating the dependency on E_s , L_- and L_+^∞ , the original reformulated SAIL++ Eq. (92) becomes

$$\frac{dE_o^+}{dz} = wE_s + \mathbf{v} \circ L_- + \mathbf{v}' \circ L_+^\infty + d_L \sum_{n=0}^{+\infty} (-1)^n A_n P_{\chi,n} [\mathbf{v}'_0 \circ L_+^{1,n}(z)] - KE_o^+. \quad (32)$$

Similarly, Eqs. (91), (90) and (93) become respectively

$$\begin{aligned} \frac{dL_-}{dz} &= -s' \circ E_s + \mathfrak{A} \circ L_- - \mathfrak{B} \circ L_+^\infty - d_L \sum_{n=0}^{+\infty} (-1)^n A_n P_{\chi,n} [\mathfrak{B}_0 \circ L_+^{1,n}(z)], \\ \frac{dL_+^\infty}{dz} &= \circ \circ E_s + \mathfrak{B} \circ L_- - \mathfrak{A} \circ L_+^\infty + d_L \sum_{n=0}^{+\infty} (-1)^n A_n P_{\chi,n} [\mathfrak{B}'_0 \circ L_+^{1,n}(z)], \\ \frac{dE_o^-}{dz} &= -w'E_s - \mathbf{v}' \circ L_- - \mathbf{v} \circ L_+^\infty - d_L \sum_{n=0}^{+\infty} (-1)^n A_n P_{\chi,n} [\mathbf{v}_0 \circ L_+^{1,n}(z)] + KE_o^-, \end{aligned} \quad (33)$$

with $\mathfrak{o}$ the vacuum operator, $\mathfrak{B}_0$, $\mathfrak{B}'_0$ and $\mathfrak{v}_0$ the normalized scattering parameters corresponding to $\mathfrak{B}$, $\mathfrak{B}'$ and $\mathfrak{v}$ [cf. Eqs. (73) (74) (75)], respectively.

3. Virtual flux decomposition implementation

As in SAIL++ (cf. Appendix A.2), the implementation needs the discretization of the diffuse fluxes over the hemispheres. These diffuse fluxes correspond to the diffuse radiances $(L_+^{1,n})_{n \in \mathbb{N}}$, L_+^∞ and L_- when only a vegetation layer is considered (cf. Subsection 2.2). The corresponding discrete fluxes will be called $(E_+^{1,n})_{n \in \mathbb{N}}$, E_+^∞ and E_- , respectively. The reflectances created by scattering of $(E_+^{1,n})_{n \in \mathbb{N}}$ and $(E_+^{0,n})_{n \in \mathbb{N}}$ will be separated to the one created by E_s . The separation enables the solution of the RT problem based on SAIL++ formalism.

First, we present the processing of the vegetation layer. Second, we show the soil vegetation coupling.

3.1 Vegetation layer

3.1.1 $E_+^{1,n}$ estimation

As reformulated in Appendix A.3, the difference between SAIL++ and our model occurs in the calculation of L_+^1 . In our model it is decomposed into the sequence $(L_+^{1,n})_{n \in \mathbb{N}}$ thus modifying the expressions of L_- , L_+^∞ , E_o^+ and E_o^- . Therefore, in this section, we propose the derivation of a new expression for the discrete fluxes E_- and E_+^∞ as well as the radiances E_o^+ and E_o^- versus $(E_+^{1,n})_{n \in \mathbb{N}}$.

Now, $\forall n \in \mathbb{N}$, $L_+^{1,n}$ is given by Eq. (20). Let us consider the Verhoef (1998) sphere tessellation into N segments, then the irradiance $E_{+,i}^{1,n}$ of each segment i is

$$\begin{aligned} E_{+,i}^{1,n}(z) &= \int_{\Delta\Omega_i} L_+^{1,n}(z, \Omega) \cos(\theta) d\Omega, \\ &\approx E_s(0) \frac{1 - \exp[-(k + \langle \kappa \rangle_{\Delta\Omega_i} + n \langle b \rangle_{\Delta\Omega_i})(H + z)]}{k + \langle \kappa \rangle_{\Delta\Omega_i} + n \langle b \rangle_{\Delta\Omega_i}} \exp(kz) \\ &\quad \times \int_{\Delta\Omega_i} \pi^{-1} w(\Omega_s \rightarrow \Omega) \cos(\theta) d\Omega, \end{aligned} \quad (34)$$

where $\langle \cdot \rangle_{\Delta\Omega_i}$ is the mean value operator defined for a given function f as follows

$$\langle f(\Omega) \rangle_{\Delta\Omega_i} = \frac{\int_{\Omega \in \Delta\Omega_i} f(\Omega) \cos(\Omega) d\Omega}{\int_{\Omega \in \Delta\Omega_i} \cos(\Omega) d\Omega}. \quad (35)$$

Following Verhoef (1998) terminology,

$$\begin{aligned} \langle \kappa \rangle_{\Delta\Omega_i} &= \kappa(i), \\ \int_{\Delta\Omega_i} \pi^{-1} w(\Omega_s \rightarrow \Omega) \cos(\theta) d\Omega &= \mathbf{s}(i), \end{aligned} \quad (36)$$

similarly, we adopt the following notation

$$\langle b \rangle_{\Delta\Omega_i} = \mathbf{b}(i), \quad (37)$$

thus κ_n [cf. Eq. (17)] will be extended in the discrete case as follows

$$\kappa_n(i) = \kappa(i) + n\mathbf{b}(i). \quad (38)$$

3.1.2 $E_{+,i}^{1,n}$ dependency

Being scattered, $E_{+,i}^{1,n}$ can create both diffuse fluxes E_+^∞ and E_- as well as radiances E_o^+ and E_o^- . The scattering parameters will be called respectively $s_{i,n}$, $s'_{i,n}$, $w'_{i,n}$ and $w_{i,n}$. Now,

$$w'_{i,n}(\Omega_o) = d_L P_{\chi,n} \langle w'_0(\Omega \rightarrow \Omega_o) \rangle_{\Omega \in \Delta\Omega_i} = P_{\chi,n} \mathbf{v}'_0(i), \quad (39)$$

where $\mathbf{v}'_0$ is the normalized SAIL++ scattering parameter corresponding to $\mathbf{v}'$ [cf. Eq. (85)]. Similarly, one can define $w_{i,n}$ the analogue of $w'_{i,n}$ when $\Delta\Omega_i$ and Ω_o are in the same hemisphere

$$w_{i,n}(\Omega_o) = d_L P_{\chi,n} \langle w_0(\Omega \rightarrow \Omega_o) \rangle_{\Omega \in \Delta\Omega_i} = P_{\chi,n} \mathbf{v}_0(i), \quad (40)$$

where $\mathbf{v}_0$ is the normalized scattering parameter corresponding to $\mathbf{v}$ [cf. Eq. (85)].

As in the SAIL++ model (Verhoef, 1998), $s_{i,n}$ and $s'_{i,n}$ are integrated values of $w_{i,n}$ and $w'_{i,n}$ over the output solid angle. So, for $m \in \{1, \dots, N\}$ a given discrete solid angle index

$$\begin{aligned} s_{i,n}(m) &= \int_{\Delta\Omega_m} w_{i,n}(\Omega_m) d\Omega_m = d_L P_{\chi,n} \pi^{-1} \langle \langle w_0(\Omega \rightarrow \Omega_+) \rangle \rangle_{(\Omega, \Omega_+) \in (\Delta\Omega_i, \Delta\Omega_m)} \frac{2\pi}{N}, \\ &= d_L P_{\chi,n} \mathbf{B}'_0(i \rightarrow m), \end{aligned} \quad (41)$$

where $\mathbf{B}'_0$ is the normalized SAIL++ scattering matrix corresponding to $\mathbf{B}'$ [cf. Eq. (86)]. Similarly,

$$s_{i,n}(m) = d_L P_{\chi,n} \mathbf{B}_0(i \rightarrow m), \quad (42)$$

where $\mathbf{B}_0$ is the normalized scattering matrix corresponding to $\mathbf{B}$ [cf. Eq. (85)].

3.1.3 $E_{+,i}^{1,n}$ decomposition

From Eq. (34), one has

$$\begin{aligned} E_{+,i}^{1,n}(z) &= E_s(0) \frac{1 - \exp[-(k + \kappa_n(i))(H + z)]}{k + \kappa_n(i)} \exp(kz) \mathbf{s}(i), \\ &= X_i^n E_{+,i,1}^{1,n}(z) + Y_i^n E_{+,i,2}^{1,n}(z). \end{aligned} \quad (43)$$

with

$$\begin{aligned} X_i^n &= \frac{\mathbf{s}(i)}{k + \kappa_n(i)}, \\ Y_i^n &= -\frac{\mathbf{s}(i) \exp(-kH)}{k + \kappa_n(i)}, \\ E_{+,i,1}^{1,n}(z) &= E_s(0) \exp(kz) = E_s(z), \\ E_{+,i,2}^{1,n}(z) &= E_s(0) \exp[-\kappa_n(i)(H + z)]. \end{aligned} \quad (44)$$

Therefore, $E_{+,i,1}^{1,n}$ and $E_{+,i,2}^{1,n}$ can be viewed as the direct downward and upward fluxes with an extinction factor under the vegetation equal to k and $\kappa_n(i)$, respectively.

Thus, the corresponding RT discrete equation set to the continuous Eqs. (32) (33) presented in the last section is

$$\left\{ \begin{array}{l} E_{+,i,1}^{1,n}(0) = E_{+,i,2}^{1,n}(-H) = E_s(0), \forall \{i,n\} \in \{1,\dots,N\} \times \mathbb{N}, \\ \frac{dE_s}{dz} = kE_s, \\ \frac{dE_{+,i,1}^{1,n}}{dz} = kE_{+,i,1}^{1,n}, \forall \{i,n\} \in \{1,\dots,N\} \times \mathbb{N}, \\ \frac{dE_{+,i,2}^{1,n}}{dz} = -\kappa_n(i)E_{+,i,2}^{1,n}, \forall \{i,n\} \in \{1,\dots,N\} \times \mathbb{N}, \\ \frac{dE_-}{dz} = -s'E_s + \mathbf{A}E_- - \mathbf{B}E_+ - \sum_{n=0}^{\infty} (-1)^n \sum_{i=1}^N A_i^n (X_i^n \mathbf{s}_{i,n} E_{+,i,1}^{1,n} + Y_i^n \mathbf{s}_{i,n} E_{+,i,2}^{1,n}), \\ \frac{dE_+^{\infty}}{dz} = \mathbf{B}E_- - \mathbf{A}E_+ + \sum_{n=0}^{\infty} (-1)^n \sum_{i=1}^N A_i^n (X_i^n \mathbf{s}'_{i,n} E_{+,i,1}^{1,n} + Y_i^n \mathbf{s}'_{i,n} E_{+,i,2}^{1,n}), \\ \frac{dE_o^+}{dz} = wE_s + \mathbf{v}E_- + \mathbf{v}'E_+ + \sum_{n=0}^{\infty} (-1)^n \sum_{i=1}^N A_i^n (X_i^n w'_{i,n} E_{+,i,1}^{1,n} + Y_i^n w'_{i,n} E_{+,i,2}^{1,n}) - KE_o^+, \\ \frac{dE_o^-}{dz} = -w'E_s - \mathbf{v}'E_- - \mathbf{v}E_+ - \sum_{n=0}^{\infty} (-1)^n \sum_{i=1}^N A_i^n (X_i^n w_{i,n} E_{+,i,1}^{1,n} + Y_i^n w_{i,n} E_{+,i,2}^{1,n}) + KE_o^-, \end{array} \right. \quad (45)$$

with A_i^n the extension of A_n to the discrete case Eq. (17)

$$A_i^n = \frac{(k\kappa(i))^{n/2}}{n! \mathbf{b}(i)^n} \exp \left[\frac{\sqrt{k\kappa(i)}}{\mathbf{b}(i)} \right]. \quad (46)$$

From a mathematical perspective, System 45 could be viewed as follows. The unknowns are E_- , E_+^{∞} , E_o^+ and E_o^- . They have to be solved using three differential equations linking them (three last Equations in Set 45). In addition to the unknown functions, the differential equations contain additive terms composed of linear combinations of known functions which are E_s and $E_{+,i,j}^{1,n}, \forall \{i,j,n\} \in \{1,\dots,N\} \times \{1,2\} \times \mathbb{N}$. Therefore, solutions to the global differential equation set (E_- , E_+^{∞} , E_o^+ and E_o^-) can be written as linear combinations (the same as the combination of the additive terms in the initial set) of the same differential equation set solutions with only one additive term among the set E_s , $E_{+,i,j}^{1,n}, \forall \{i,j,n\} \in \{1,\dots,N\} \times \{1,2\} \times \mathbb{N}$.

Therefore, we propose the following solution. E_- , E_+^{∞} , E_o^+ and E_o^- have to be derived for different sources: $E_s(0)$, $E_{+,i,1}^{1,n}(0)$ and $E_{+,i,2}^{1,n}(-H)$, $\forall \{i,n\} \in \{1,\dots,N\} \times \mathbb{N}$. For that, one can define the corresponding sub-solutions which are E_-^s , $E_+^{\infty,s}$, $E_o^{+,s}$, $E_o^{-,s}$, $\forall \{i,j,n\} \in \{1,\dots,N\} \times \{1,2\} \times \mathbb{N}$, $E_{-,i,j}^n$, $E_{+,i,j}^{\infty,n}$, $E_{o,i,j}^{+,n}$ and $E_{o,i,j}^{-,n}$, respectively.

According to Eqs. (45), the global solution for $E \in \{E_-, E_+^{\infty}, E_o^+, E_o^-\}$ is written as follows

$$E = E^s + \sum_{n=0}^{\infty} (-1)^n \sum_{i=1}^N A_i^n (X_i^n E_{i,1}^n + Y_i^n E_{i,2}^n), \quad (47)$$

Now, compared to SAIL++ in terms of boundary conditions (cf. Appendix A.2), each term x of the boundary condition matrix [cf. Eq. (87)] that depends on the direct source flux [cf. Eq.

(88)], i.e.

$$x \in \left\{ \tau_{ss} = \frac{E_s(-H)}{E_s(0)}, \tau_{sd} = \frac{E_-(-H)}{E_s(0)}, \rho_{sd} = \frac{E_+^0(0) + E_+^\infty(0)}{E_s(0)}, \rho_{so} = \frac{E_o^+(0)}{E_s(0)}, \tau_{so} = \frac{E_o^-(0)}{E_s(0)} \right\} \quad (48)$$

has to be modified. The other boundary matrix terms ($T, R, \tau_{do}, \rho_{do}$ and τ_{oo}) remain equivalent to SAIL++.

Moreover, ρ_{sd} is divided into two terms

$$\begin{aligned} \rho_{sd} &= \rho_{sd}^1 + \rho_{sd}^\infty, \\ \rho_{sd}^1 &= \frac{E_+^0(0)}{E_s(0)}, \\ \rho_{sd}^\infty &= \frac{E_+^\infty(0)}{E_s(0)}. \end{aligned} \quad (49)$$

In the case of $x \in \{\tau_{ss}, \tau_{sd}, \rho_{sd}^\infty, \rho_{so}, \tau_{so}\}$ and according to Eqs. (47)

$$x = x^s + \sum_{n=0}^{\infty} (-1)^n \sum_{i=1}^N A_i^n (X_i^n x_{i,1}^n + Y_i^n x_{i,2}^n), \quad (50)$$

with x^s the value corresponding to the source E_s , and $\forall \{i, j, n\} \in \{1, \dots, N\} \times \{1, 2\} \times \mathbb{N}$, $x_{i,j}^n$ the value corresponding to the source $E_{+,i,j}^{n,1}$. Based on Eqs. (43) (44)

$$\forall i \in \{1, \dots, N\}, \rho_{sd}^1(i) = \sum_{n=0}^{\infty} (-1)^n A_i^n (X_i^n + Y_i^n \exp[-\kappa_n(i)H]). \quad (51)$$

Note that τ_{ss} , the direct transmittance, does not change, it is equal to $\exp(-kH)$. Therefore, we have to derive only $\tau_{sd}, \rho_{sd}^\infty, \rho_{so}$ and τ_{so} .

3.1.4 Sub-solution derivation

Here, we try to find the sub-solution scattering term expressions ($\tau_{sd}, \rho_{sd}^\infty, \rho_{so}$ and τ_{so}) based on SAIL++ formalism and versus SAIL++ boundary matrix terms. To achieve it, the irradiance E_+^∞ , E_- and radiances E_o have to be estimated. The latter terms have first to be estimated for each source among E_s and $\forall \{i, j, n\} \in \{1, \dots, N\} \times \{1, 2\} \times \mathbb{N}$, $E_{-,i,j}^n$ and second combined using Eq. (47).

3.1.4.1 Source E_s

$E_-^s, E_+^{\infty,s}, E_o^{+,s}$ and $E_o^{-,s}$ verify

$$\frac{d}{dz} \begin{pmatrix} E_s \\ E_-^s \\ E_+^{\infty,s} \\ E_o^{+,s} \\ E_o^{-,s} \end{pmatrix} = \begin{pmatrix} k & 0 & 0 & 0 & 0 \\ -s' & \mathbf{A} & -\mathbf{B} & 0 & 0 \\ 0 & \mathbf{B} & -\mathbf{A} & 0 & 0 \\ w & \mathbf{v} & \mathbf{v}' & -K & 0 \\ -w' & -\mathbf{v}' & -\mathbf{v} & 0 & K \end{pmatrix} \begin{pmatrix} E_s \\ E_-^s \\ E_+^{\infty,s} \\ E_o^{+,s} \\ E_o^{-,s} \end{pmatrix}, \quad (52)$$

Thus based on Eq. (88) notation, it follows

$$\begin{aligned} \tau_{sd}^s &= \tau_{sd}^{++}(k, s', 0), \\ \rho_{sd}^{s,\infty} &= \rho_{sd}^{++}(k, s', 0), \\ \rho_{so}^s &= \rho_{so}^{HS,++}(k, s', 0, w), \\ \tau_{so}^s &= \tau_{so}^{++}(k, s', 0, w). \end{aligned} \quad (53)$$

3.1.4.2 Source $E_{+,i,1}^{1,n}$

As for E_s [cf. Eq. (53)], it is straightforward to show that

$$\begin{aligned}\tau_{sd,i,1}^n &= \tau_{sd}^{++}(k, \mathbf{s}_{i,n}, \mathbf{s}'_{i,n}), \\ \rho_{sd,i,1}^n &= \rho_{sd}^{++}(k, \mathbf{s}_{i,n}, \mathbf{s}'_{i,n}), \\ \rho_{so,i,1}^n &= \rho_{so}^{++}(k, \mathbf{s}_{i,n}, \mathbf{s}'_{i,n}, w'_{i,n}), \\ \tau_{so,i,1}^n &= \tau_{so}^{++}(k, \mathbf{s}_{i,n}, \mathbf{s}'_{i,n}, w'_{i,n}).\end{aligned}\quad (54)$$

3.1.4.3 Source $E_{+,i,2}^{1,n}$

As for E_s [cf. Eq. (53)], it is straightforward to show that

$$\begin{aligned}\tau_{sd,i,2}^n &= \rho_{sd}^{++}(\kappa_n(i), \mathbf{s}'_{i,n}, \mathbf{s}_{i,n}), \\ \rho_{sd,i,2}^n &= \tau_{sd}^{++}(\kappa_n(i), \mathbf{s}'_{i,n}, \mathbf{s}_{i,n}), \\ \rho_{so,i,2}^n &= \tau_{so}^{++}(\kappa_n(i), \mathbf{s}'_{i,n}, \mathbf{s}_{i,n}, w'_{i,n}), \\ \tau_{so,i,2}^n &= \rho_{so}^{++}(\kappa_n(i), \mathbf{s}'_{i,n}, \mathbf{s}_{i,n}, w'_{i,n}).\end{aligned}\quad (55)$$

Finally, according to Eqs. (50) (53) (54) (55)

$$\begin{aligned}\tau_{sd} &= \tau_{sd}^{++}(k, \mathbf{s}', 0) + \sum_{n=0}^{\infty} (-1)^n \sum_{i=1}^N A_i^n (X_i^n \tau_{sd}^{++}(k, \mathbf{s}_{i,n}, \mathbf{s}'_{i,n}) + Y_i^n \rho_{sd}^{++}(\kappa_n(i), \mathbf{s}'_{i,n}, \mathbf{s}_{i,n})), \\ \rho_{sd}^{\infty} &= \rho_{sd}^{++}(k, \mathbf{s}', 0) + \sum_{n=0}^{\infty} (-1)^n \sum_{i=1}^N A_i^n (X_i^n \rho_{sd}^{++}(k, \mathbf{s}_{i,n}, \mathbf{s}'_{i,n}) + Y_i^n \tau_{sd}^{++}(\kappa_n(i), \mathbf{s}'_{i,n}, \mathbf{s}_{i,n})),\end{aligned}\quad (56)$$

$$\begin{aligned}x_{so} &= \rho_{so}^{HS,++}(k, \mathbf{s}', 0, w) + \sum_{n=0}^{\infty} (-1)^n \sum_{i=1}^N A_i^n (X_i^n \rho_{so}^{++}(k, \mathbf{s}_{i,n}, \mathbf{s}'_{i,n}, w'_{i,n}) + \\ &\quad Y_i^n \tau_{so}^{++}(\kappa_n(i), \mathbf{s}'_{i,n}, \mathbf{s}_{i,n}, w'_{i,n})), \\ \tau_{so} &= \tau_{so}^{++}(k, \mathbf{s}', 0, w) + \sum_{n=0}^{\infty} (-1)^n \sum_{i=1}^N A_i^n (X_i^n \tau_{so}^{++}(k, \mathbf{s}_{i,n}, \mathbf{s}'_{i,n}, w'_{i,n}) + \\ &\quad Y_i^n \rho_{so}^{++}(\kappa_n(i), \mathbf{s}'_{i,n}, \mathbf{s}_{i,n}, w'_{i,n})).\end{aligned}\quad (57)$$

3.2 Concatenation vegetation layer and soil background

Similarly to vegetation, one can define the directional-hemispherical reflectance ($\mathbf{r}_{sd}$), hemispherical-directional reflectance ($\mathbf{r}_{do}$) and hemispherical-hemispherical reflectance ($\mathbf{R}_{dd}$) for soil which are two vectors and a matrix, respectively.

Based on Adding principle (Van de Hulst, 1980), Verhoef (1998) defines the bidirectional reflectance of the couple soil+vegetation (ρ_{so}^t) as

$$\rho_{so}^t = \rho_{so} + \tau_{oo} r_{so} \tau_{ss} + (\tau_{oo} \mathbf{r}_{do}^T + \tau_{do}^T \mathbf{R}_{dd})(1 - \mathbf{R} \mathbf{R}_{dd})^{-1} \tau_{sd} + (\tau_{do}^T + \tau_{oo} \mathbf{r}_{do}^T \mathbf{R})(I - \mathbf{R}_{dd} \mathbf{R})^{-1} \mathbf{r}_{sd} \tau_{ss}, \quad (58)$$

with I the identity matrix.

Inspired from AddingSD (e.g. Kallel et al., 2008, p. 3647), we propose the following transformation of Eq. (58)

$$\rho_{so}^t = \rho_{so} + \overbrace{\tau_{oo} r_{so} \tau_{ss}}^{r_{sso}} + \overbrace{\tau_{do}^T \mathbf{r}_{sd} \tau_{ss}}^{r_{sdo}} + (\tau_{do}^T \mathbf{R}_{dd} + \tau_{oo} \mathbf{r}_{do}^T)(I - \mathbf{R} \mathbf{R}_{dd})^{-1} \overbrace{(\mathbf{R} \mathbf{r}_{sd} \tau_{ss} + \tau_{sd})}^{\tau_{sdd}}. \quad (59)$$

As rigorously explained in (Kallel et al., 2008), to pass from a turbid to a discrete case and take into account the hot spot effect as well as maintain energy conservation, we have to modify the expression of r_{ss0} , r_{sdo} and τ_{sdd} by considering the actual local vegetation density:

- r_{ss0} corresponds to the flux passing through the vegetation layer from top to bottom without collision, scattered by the soil and reaching the top of the vegetation without other collisions. For this flux, the classical hot spot effect should be computed as

$$r_{ss0} = r_{so} \exp \left[-(k + K)H + \frac{\sqrt{kK}}{b} [1 - \exp(-bH)] \right]; \quad (60)$$

- τ_{sdd} corresponds to the flux passing through the vegetation layer from top to bottom without collisions, scattered by the soil, colliding with the vegetation and reaching the soil again.
- r_{sdo} corresponds to the flux passing through the vegetation layer from top to bottom without collisions, scattered by the soil and reaching the top of the vegetation after multiple collisions.

Using the same principle that for E_+^1 scattering derivation, it is straightforward to show that

$$\begin{aligned} \tau_{sdd} &= \exp(-kH) \sum_{n=0}^{\infty} (-1)^n \sum_{i=1}^N A_i^n \mathbf{a}(i) \rho_{sd}^{++}(\kappa_n(i), \mathbf{s}'_{i,n}, \mathbf{s}_{i,n}), \\ r_{sdo} &= \exp(-kH) \sum_{n=0}^{\infty} (-1)^n \sum_{i=1}^N A_i^n \mathbf{a}(i) \tau_{so}^{++}(\kappa_n(i), \mathbf{s}'_{i,n}, \mathbf{s}_{i,n}, w_{i,n}). \end{aligned} \quad (61)$$

4. Virtual flux decomposition validation

This section is dedicated to the validation of our virtual flux decomposition. The corresponding will be called the Flux Decomposition Model (FDM). First, model convergence and running time are presented. Second, energy conservation is shown. Third, a comparison between our approach and SAIL/SAIL++ models is presented. Finally, our model is compared to the 3-D models of the RAMI 2 database assumed 'most credible'.

Among the most commonly used models to describe the distribution of leaf zenithal angles is the method assuming an elliptic leaf distribution where the distribution is parameterized by the mean leaf inclination angle, ALA , ranging between 0 and 90° (Campbell, 1990). We will use this distribution in our model simulations. Note that small ALA values correspond to planophile distributions, high values to erectophile distributions, and medium values to extremophile distributions. Moreover, to be compatible with RAMI database simulations, Bunnik's [1978] parametrization will be used in the fourth subsection.

Since FDM is equivalent to SAIL++ in the turbid case. In this paper, we will deal only with the discrete case.

4.1 Running time

Among the strengths of our model is its low running time. The decomposition of L_+^1 into virtual sub-fluxes allowed the use of SAIL++ formalism to solve the RT problem.

Although, according to Eqs. (56) (57) (61), an infinite number of SAIL++ simulations is needed to derive the reflectance, only few first ranked terms are used to achieve accurate results. The sum is provided by Taylor series decomposition. Next, we will present a study on the accuracy of the approximation.

Here, we opt to use fluxes (E_+^∞ in our case) and the corresponding hemispherical scattering (ρ_{sd}^∞) term rather than radiances E_o^+ or E_o^- . For energy conservation, it is more significant to deal with fluxes than their densities.

Recall that

$$\begin{aligned} E_+^\infty &= E_+^{\infty,s} + \sum_{n=0}^{\infty} (-1)^n \sum_{i=1}^N A_i^n (X_i^n E_{+,ij}^{\infty,n} + Y_i^n E_{+,ij}^{\infty,n}) = E_+^{\infty,s} + \sum_{n=0}^{\infty} (-1)^n \sum_{i=1}^N A_i^n E_{+,i}^{\infty,n}, \\ &= E_+^{\infty,s} + \sum_{i=1}^N \sum_{n=0}^{\infty} (-1)^n A_i^n E_{+,i}^{\infty,n}, \end{aligned} \quad (62)$$

where $E_{+,i}^{\infty,n}$ is the high order diffuse flux created by $E_{+,i}^{1,n}$. Now, according to Eq. (22), $E_{+,i}^{1,n}$ is created by E_s scattering. Then, due to the energy conservation law

$$\forall (z, i) \in [-H, 0] \times \{1, \dots, N\}, \|E_{+,i}^{\infty,n}(z)\| \leq E_s(0), \quad (63)$$

where $\|\cdot\|$ of a given discrete flux over a hemisphere is the sum of the sub-fluxes' values in each segment. It corresponds to the integrate radiance distribution over the hemisphere.

Let us assume that, $\forall i \in \{1, \dots, N\}$, the series $\sum_{n=0}^{\infty} (-1)^n A_i^n E_{+,i}^{\infty,n}$ is truncated to the order u_i . Let us define a vector $\mathbf{u}$ by

$$\mathbf{u} = [u_1, \dots, u_N], \quad (64)$$

and the corresponding flux $E_+^{\infty,\mathbf{u}}$

$$E_+^{\infty,\mathbf{u}} = E_+^{\infty,s} + \sum_{i=1}^N \sum_{n=0}^{u_i} (-1)^n A_i^n E_{+,i}^{\infty,n}, \quad (65)$$

then

$$\begin{aligned} \|E_+^\infty - E_+^{\infty,\mathbf{u}}\| &= \left\| \sum_{i=1}^N \sum_{n=u_i+1}^{\infty} (-1)^n A_i^n E_{+,i}^{\infty,n} \right\| \leq \sum_{i=1}^N \sum_{n=u_i+1}^{\infty} A_i^n \|E_{+,i}^{\infty,n}\| \leq \sum_{i=1}^N \sum_{n=u_i+1}^{\infty} A_i^n E_s(0), \\ &\leq E_s(0) \sum_{i=1}^N \exp \left[\frac{\sqrt{k\kappa(i)}}{b(i)} \right] \sum_{n=u_i+1}^{\infty} \frac{(k\kappa(i))^{n/2}}{n! b(i)^n}, \\ &\leq E_s(0) \sum_{i=1}^N \exp \left[\frac{\sqrt{k\kappa(i)}}{b(i)} \right] \frac{(k\kappa(i))^{(u_i+1)/2}}{(u_i+1)! b(i)^{u_i+1}} \sum_{n=0}^{\infty} \frac{(k\kappa(i))^{n/2} (u_i+1)!}{(n+u_i+1)! b(i)^n}, \\ &\leq E_s(0) \sum_{i=1}^N \exp \left[\frac{\sqrt{k\kappa(i)}}{b(i)} \right] \frac{(k\kappa(i))^{(u_i+1)/2}}{(u_i+1)! b(i)^{u_i+1}} \sum_{n=0}^{\infty} \frac{(k\kappa(i))^{n/2}}{n! b(i)^n}, \\ &\leq E_s(0) \underbrace{\sum_{i=1}^N \exp \left[\frac{\sqrt{k\kappa(i)}}{b(i)} \right] A_i^{u_i+1}}_{B_i^{u_i+1}}. \end{aligned} \quad (66)$$

It is clear that, $\lim_{u_i \rightarrow +\infty} B_i^{u_i+1} = 0$, then

$$\lim_{u_1 \rightarrow +\infty} \lim_{u_2 \rightarrow +\infty} \dots \lim_{u_N \rightarrow +\infty} \|E_+^\infty - E_+^{\infty,\mathbf{u}}\| = 0. \quad (67)$$

Moreover, let us define $\rho_{sd}^{\infty, \mathbf{u}}$ by

$$\rho_{sd}^{\infty, \mathbf{u}} = \frac{E_+^{\infty, \mathbf{u}}}{E_s(0)}, \quad (68)$$

then according to Eq. (66),

$$\|\rho_{sd}^{\infty} - \rho_{sd}^{\infty, \mathbf{u}}\| \leq \sum_{i=1}^N B_i^{u_i+1}. \quad (69)$$

Therefore, to ensure that $\|\rho_{sd}^{\infty} - \rho_{sd}^{\infty, \mathbf{u}}\| \leq \varepsilon$ with $\varepsilon \in \mathbb{R}_+^*$ such that $\varepsilon \ll 1$, it is sufficient to choose $\mathbf{u}$ as follows,

$$\forall i \in \{1, \dots, N\}, u_i = \operatorname{argmin}_{n \in \mathbb{N}} B_i^{n+1} \leq \frac{\varepsilon}{N}. \quad (70)$$

Fig. 2. Variation of a) the mean $\mathbf{u}$ value and b) the corresponding model running time as a function of LAI and the hot spot parameter d for a source zenith angle $\theta_s = 25^\circ$, $ALA = 63^\circ$, $\rho = 0.5$, $\tau = 0.5$ and a model error threshold $\varepsilon = 10^{-4}$.

Figure 2 shows the variation of the average $\mathbf{u}$ value ($\langle \mathbf{u} \rangle$) as well as the corresponding model running time, T , versus the Leaf Area Index value, LAI, and the hot spot parameter (ratio between the leaf mean radius and the vegetation layer thickness), d , for $\varepsilon = 10^{-4}$. As expected, $\langle \mathbf{u} \rangle$ and the corresponding T value increases as a function of LAI and d . $\langle \mathbf{u} \rangle$ ranges between 1 and 10 which is a relatively high speed convergence. For medium LAI and d values (respectively equal 3 and 0.05), $\langle \mathbf{u} \rangle$ is about 4.2. This speed convergence explains the small running time for such a complex model: it ranges between 2 and 22 milliseconds (ms). In particular, for LAI=3 and $d = 0.05$, it is about 10 ms .

4.2 Energy conservation

Compared to SAIL++, the advantage of our model is energy conservation in the discrete case. To check conservation, we propose to use the same procedure as Kallel et al. (2008). In the purist corner case ($\rho + \tau = 1$), the energy conservation law is

$$B(\Omega_s) = \tau_{ss} + \int_{\Pi} \{\rho_{so}(\Omega_o) + \tau_{so}(\Omega_o)\} \cos(\theta_o) d\Omega_o = 1. \quad (71)$$

Figure 3 shows the variation of B versus LAI, for extremophile and erectophile vegetation (ALA equals respectively 45° and 63°) and for different hot spot parameters. In both cases,

Fig. 3. Variation of energy conservation (B) versus LAI for different values of the hot spot parameter (d), $\theta_s = 25^\circ$, $\rho = 0.5$ and $\tau = 0.5$.

for small LAI values, the energy is well conserved by both models with an error lower than 0.1%. However, for values of LAI ranging from 0.5 to 1, B decreases to around 0.99 for both cases. The decrease of accuracy is due to the sampling of the hemisphere in only 36 segments and assuming a constant radiance distribution over each segment. The decrease is more pronounced for erectophile leaf distributions since the validity of the diffuse flux isotropy assumption weakens as ALA increases (Kallel et al., 2008). An increase in the segment number extends the running time. According to Verhoef (1998), 36 segments is a trade-off between accuracy and running time. Reaching its minimum values, B increases versus LAI in different ways for different models. First, for $d = 0.01$, the increase is relatively small and energy is accurately conserved by both models. Second, for $d = 0.05$ and $d = 0.1$, Figure 3 shows large differences between models. SAIL++ conserves energy less. For example for LAI=7 and $ALA = 45^\circ$, the variation in energy conservation (B) was 1.04 whereas for our model it was less than 1.015. This proves our main objective for the new model: energy conservation. Moreover, comparing extremophile and erectophile cases, Figure 3 shows that energy is better conserved in the erectophile case. Indeed, mutual shadowing between leaves decreases as a function of ALA , and thus it is higher in the extremophile case.

4.3 Model comparison: SAIL/SAIL++

In this subsection, BRDF and BTDF produced by our model are compared to ones produced by the SAIL and SAIL++ models in the discrete case.

Figure 4 shows the case of a hot spot parameter ($d = 0.02$) for extremophile ($ALA = 45^\circ$) and erectophile vegetation ($ALA = 63^\circ$) in the principle plane. First, figures show the hot spot peaks for $\theta_o = \theta_s$ and $\varphi = 0^\circ$. Second, when a soil background is added, the model reflectances increase but the curve dynamics decrease due to soil lambertianity. Third, since SAIL underestimates the reflectance due to the diffuse flux semi-isotropy assumption (Kallel, 2007), its BRDF curves are always below SAIL++ curves. Fourth, compared to SAIL++, our model's BRDF and BTDF curves are always lower: our model decreases SAIL++ diffuse fluxes E_+^∞ and

Fig. 4. A comparison between BRDF of SAIL, SAIL++ and FDM versus the observation zenith angle for (i) only a vegetation layer (noted in the legend BRDF v); (ii) for a vegetation layer+lambertian soil (noted in the legend BRDF v/s), and BTDF of SAIL++ and FDM for a vegetation layer. $LAI=3$, $d = 0.1$, $\theta_s = 25^\circ$, $\rho = 0.5$, $\tau = 0.5$ and $r_{so} = 0.2$.

E_- in order to conserve the energy. Now, let us compare the extremophile and erectophile cases. (i) Extremophile vegetation has a higher reflectance (and a lower transmittance) than the erectophile one. Indeed, interception is higher for more vertical vegetation; (ii) SAIL and SAIL++ curves are closer to each other in the extremophile case because the semi-isotropic diffuse flux assumption is more conserved in this case (Kallel et al., 2008); (iii) FDM curves are closer to SAIL++ in the erectophile vegetation case than the extremophile case because the mutual shadowing decreases as ALA increases. Thus, the hot spot effect is less pronounced for erectophile vegetation. This was also seen in Subsection 4.2: for SAIL++ the energy is less conserved in the extremophile case similarly to Figure 3 where the hot spot peak is more narrow in the erectophile case; (iv) Finally, although SAIL underestimates the reflectance, due to the high hot spot effect, FDM curves are lower than SAIL curves in the extremophile case.

4.4 Model comparison: RAMI database

The RAdiation transfer Model Intercomparison (RAMI) exercise (Pinty et al., 2001; 2004) proposes some protocols to benchmark radiative transfer models applied to plant canopies covering soil surfaces. The object of RAMI is to validate the reliability and accuracy of different models in simulating RT in or near a vegetation canopy for the benefit of remote sensing data interpretation.

The present study only deals with homogenous vegetation assumed a discrete medium. Also, we only present the BRDF relative to the near-infrared domain, since, in this case, the leaf albedo is higher than the other wavelength cases and, thus, the corresponding simulation results have larger contrasts. In the RAMI exercise second phase (Pinty et al., 2004), the two types of RT models were considered: 1-D models, namely SAIL, SAIL++, 1/2 Discrete (Gobron et al., 1997), and 3-D models, namely Flight (North, 1996), DART (Gastellu-Etchegorry et al., 1996), Sprint-2 (Thompson & Goel, 1998), Raytran (Govaerts & Verstraete, 1998), RGM (Qin & Sig, 2000) and Drat (Lewis, 1999). In addition to these models, AddingSD and FDM will be shown in this Section to be compared with the others. RAMI 2 recommended using simulation

results of 3-D models as a reference for homogeneous canopies when ground truth data is not available. Moreover, simulation results of Flight, Raytran and Sprint-2 were assumed the 'most credible models'. Therefore, only the 'most credible' 3-D models will be shown in the figures.

Fig. 5. Canopy BRDF simulations for a discrete medium, at the principal plane. The vegetation features are $LAI=3$, $H = 2$, leaf radius = 0.05, erectophile leaf distribution, $\rho = 0.4957$ and $\tau = 0.4409$. The soil is assumed lambertian with reflectance equal to 0.159.

Figure 5 shows the BRDF simulations in the principal plane. In all cases, the FDM curves are close to the 3-D model ones. This proves the validity of our approach. Moreover, SAIL++ gives results close to FDM because the hot spot parameter is small ($d = \text{leaf radius}/H = 0.25$). FDM and AddingSD perform similarly since both models overcome the isotropy assumption and conserve energy. However, AddingSD curves are slightly higher than the FDM ones. This can be explained by two phenomena: (i) AddingSD describes better the multiple scattering under the vegetation since the hemispheres were decomposed into 400 segments, whereas in our case they were decomposed into only 36 segments; (ii) AddingSD takes into account the multi hot spot effect which increases the high order reflectances [cf. Fig 13, p. 3652 of (Kallel et al., 2008)]. Finally, as already shown in the previous subsection, SAIL underestimates the reflectance due to the semi-isotropy assumption.

Furthermore, for quantitative comparison with 3-D models, we already submitted our simulations to RAMI administrator to participate to the forth phase.

5. Conclusion

The goal of this chapter was to derive new methods for computing canopy reflectance so that it both conserves energy and surmounts the assumption that diffuse fluxes E_+/E_- are isotropically distributed over hemispheres. To achieve this object, we first proposed the AddingS/AddingSD models based on SAIL and Adding method and allowing to conserve energy based on the effective vegetation density approach. Second, due to the long running time of such an approach we proposed to benefited from two models: (i) SAIL++ that overcomes

the isotropy assumption and is very fast since it is based on Discrete Cosine Transformation. However, this model does not conserve energy in the hot spot; (ii) AddingSD which also overcomes the isotropy assumption and allows to conserve energy. Our new model was based on injecting the effective vegetation density approach in SAIL++, and therefore, benefited from both energy conservation and a small running time. The procedure was as follows. First, the flux created by direct solar light scattering upwards from vegetation, E_+^1 , was computed by taking into account the hot spot effect. Second, according to the effective density approach, the hot spot effect corresponded to a local vegetation density variation. Therefore, the diffuse fluxes and radiances in the observation direction (created by E_+^1) scattering have had to be estimated using the same density achieving consequently energy conservation. Third, since the computation in the latter step was too laborious, E_+^1 was decomposed into virtual subfluxes using Taylor series. Such subfluxes have a simpler expression, and were interpreted each one as a virtual direct solar flux. The provided fluxes, radiances and BRDF/BTDF by the virtual flux scattering were estimated using the SAIL++ formalism. Finally, the total model BRDF/BTDF were estimated by summing the contributions of the subfluxes.

The convergence of the Taylor series decomposition was studied. We showed that only the few first elements were needed to obtain accurate results and a small running time (ranging from 2 to 20 milliseconds). In addition, energy conservation was checked; our model showed good results with an error lower than 2%. Compared to SAIL++, our model's BRDF/BTDF curves were always lower since it decreases high order diffuse fluxes. Finally, compared to 3-D models of RAMI II database, our model gave close results.

Future research will deal with: (i) extending our model to the heterogenous medium case using the Forest Reflectance and Transmittance (FRT) Model (Kuusk & Nilson, 2000); (ii) then validating our model using the multiangular forest reflectance ground truth database described by Kuusk et al. (2008).

A. SAIL++ model

A.1 Continuous case

The formalism shown in this appendix is not presented in the original SAIL++ reference (Verhoef, 1998). However, it is needed in our study to derive the equations of our model.

The SAIL++ equations are written in the continuous case as

$$\frac{dE_s(z, \Omega_s)}{dz} = kE_s(z, \Omega_s), \quad (72)$$

$$\frac{dL_-(z, \Omega_-)}{dz} = -[\mathfrak{s}' \circ E_s(z, \Omega_s)](\Omega_-) + [\mathfrak{A} \circ L_-(z)](\Omega_-) - [\mathfrak{B} \circ L_+(z)](\Omega_-), \quad (73)$$

$$\frac{dL_+(z, \Omega_+)}{dz} = [\mathfrak{s} \circ E_s(z, \Omega_s)](\Omega_+) + [\mathfrak{B} \circ L_-(z)](\Omega_+) - [\mathfrak{A} \circ L_+(z)](\Omega_+), \quad (74)$$

$$\frac{dE_o^+(z, \Omega_o)}{dz} = wE_s(z, \Omega_s) + [\mathfrak{v} \circ L_-(z)] + [\mathfrak{v}' \circ L_+(z)] - KE_o^+(z, \Omega_o), \quad (75)$$

$$\frac{dE_o^-(z, \Omega_o)}{dz} = -w'E_s(z, \Omega_s) - [\mathfrak{v}' \circ L_-(z)] - [\mathfrak{v} \circ L_+(z)] + KE_o^-(z, \Omega_o), \quad (76)$$

where E_s is the solar incident flux, L_- , L_+ are the downward and upward hemispherical distributions of diffuse radiance, E_o^+ , E_o^- are the upward and downward radiances in the source direction times k , $\mathfrak{s}$, $\mathfrak{s}'$, $\mathfrak{A}$, $\mathfrak{B}$, w , w' , $\mathfrak{v}$, $\mathfrak{v}'$ and K are the generalized Suits scattering terms (Suits, 1972). These parameters are estimated for a given solar and sensor orientation,

leaf distribution and foliage density. The foliage area volume density (FAVD) will be called $\bar{u}_L$ (cf. Appendix B) and each Suits parameter (X) is written as follows

$$X = \bar{u}_L X_o, \quad (77)$$

with X_o a scattering parameter depending only on the leaf distribution and the solar and sensor orientations, but not the foliage density. X_o will be called the normalized parameter corresponding to X .

Applied to E_s , $\mathfrak{s}$ and $\mathfrak{s}'$ give respectively

$$\begin{aligned} [\mathfrak{s} \circ E_s(\Omega_s)](\cdot) &= \pi^{-1} w(\Omega_s \rightarrow \cdot) E_s, \\ [\mathfrak{s}' \circ E_s(\Omega_s)](\cdot) &= \pi^{-1} w'(\Omega_s \rightarrow \cdot) E_s, \end{aligned} \quad (78)$$

where w and w' are respectively the bidirectional scattering terms when the input and the output flux directions are in the same and opposite hemispheres.

Applied to a radiance distribution (L), the operator $\mathfrak{B}$ gives

$$[\mathfrak{B} \circ L](\cdot) = \pi^{-1} \int_{\Pi} w'(\Omega \rightarrow \cdot) L(\Omega) \cos(\Omega) d\Omega. \quad (79)$$

$\mathfrak{A}$ can be divided into two terms

$$\mathfrak{A} = \mathfrak{k} - \mathfrak{B}', \quad (80)$$

where $\mathfrak{k}$ and $\mathfrak{B}'$ are respectively the extinction and the scattering terms.

Since, the extinction of L_- is given by

$$\frac{dL_-(z, \Omega_-)}{dz} = \kappa L_-(z, \Omega_-). \quad (81)$$

Therefore, applied to a radiance distribution L , the operator $\mathfrak{k}$ can be defined as

$$[\mathfrak{k} \circ L](\Omega' = (\theta', \varphi')) = \int_{\Pi} \frac{\delta(\theta') \delta(\varphi')}{\cos(\theta') \sin(\theta')} \kappa(\Omega) L(\Omega) \cos(\theta) d\Omega. \quad (82)$$

Applied to a radiance distribution (L), $\mathfrak{B}'$ gives

$$[\mathfrak{B}' \circ L](\cdot) = \pi^{-1} \int_{\Pi} w(\Omega \rightarrow \cdot) L(\Omega) \cos(\Omega) d\Omega. \quad (83)$$

Applied to a radiance distribution (L), $\mathfrak{v}$ and $\mathfrak{v}'$ give respectively

$$\begin{aligned} \mathfrak{v} \circ L &= \int_{\Pi} w(\Omega \rightarrow \Omega_o) L(\Omega) \cos(\Omega) d\Omega, \\ \mathfrak{v}' \circ L &= \int_{\Pi} w'(\Omega \rightarrow \Omega_o) L(\Omega) \cos(\Omega) d\Omega. \end{aligned} \quad (84)$$

A.2 Discretization

Next, concerning the implementation and in order to describe the upward and downward diffuses radiance hemispherical distribution, Verhoef (1998) proposes a discretization of hemispheres: zenithal and azimuthal angles into N segments. In this case, L_- and L_+ are replaced by sub-fluxes defined over the hemisphere segments forming together vectors called E_- and E_+ , respectively. The operators of Eq. (72) are discretized accordingly, in particular, $\mathfrak{s}$, $\mathfrak{s}'$ become vectors called $\mathbf{s}$ and $\mathbf{s}'$, respectively, $\mathfrak{A}$, $\mathfrak{B}$ becomes square matrices called $\mathbf{A}$ and $\mathbf{B}$, respectively, and $\mathfrak{v}$ and $\mathfrak{v}'$ become vectors called $\mathbf{v}$ and $\mathbf{v}'$, respectively. Eqs. (72) (73) (74) (75) (76) become (Verhoef, 1998):

$$\frac{d}{dz} \begin{pmatrix} E_s \\ E_- \\ E_+ \\ E_o^+ \\ E_o^- \end{pmatrix} = \begin{pmatrix} k & 0 & 0 & 0 & 0 \\ -\mathbf{s}' & \mathbf{A} & -\mathbf{B} & 0 & 0 \\ \mathbf{s} & \mathbf{B} & -\mathbf{A} & 0 & 0 \\ w & \mathbf{v} & \mathbf{v}' & -K & 0 \\ -w' & -\mathbf{v}' & -\mathbf{v} & 0 & K \end{pmatrix} \begin{pmatrix} E_s \\ E_- \\ E_+ \\ E_o^+ \\ E_o^- \end{pmatrix}, \quad (85)$$

Note that, as in the continuous case [cf. Eq. (80)], $\mathbf{A}$ could be written as

$$\mathbf{A} = \boldsymbol{\kappa} - \mathbf{B}'. \quad (86)$$

with $\boldsymbol{\kappa}$ and $\mathbf{B}'$ the discrete scattering matrices corresponding to $\mathfrak{k}$ and $\mathfrak{B}'$, respectively. The final solution linking the layer output fluxes to the input ones is (Verhoef, 1998)

$$\begin{pmatrix} E_s(L) \\ E_-(L) \\ E_+(t) \\ E_o^+(t) \\ E_o^-(L) \end{pmatrix} = \begin{pmatrix} \tau_{ss} & 0 & 0 & 0 & 0 \\ \boldsymbol{\tau}_{sd} & \mathbf{T} & \mathbf{R} & 0 & 0 \\ \boldsymbol{\rho}_{sd} & \mathbf{R} & \mathbf{T} & 0 & 0 \\ \rho_{so} & \boldsymbol{\rho}_{do}^T & \boldsymbol{\tau}_{do}^T & \tau_{oo} & 0 \\ \tau_{so} & \boldsymbol{\tau}_{do}^T & \boldsymbol{\rho}_{do}^T & 0 & \tau_{oo} \end{pmatrix} \begin{pmatrix} E_s(t) \\ E_-(t) \\ E_+(L) \\ E_o^+(L) \\ E_o^-(L) \end{pmatrix}, \quad (87)$$

where (L) and (t) refer to the bottom and top of the layer, respectively.

Now, let us consider the case when the source changes. This change includes both the direction and the way that the direct flux is scattered under the vegetation. Since the scattering properties depend only on the vegetation parameters and the source solid angle, the latter possibility of change does not have a physical meaning. However, it is needed in our case to define the scattering parameter when an effective vegetation density is considered. The variation has an impact over the scattering parameters of Eq. (85) as follows. The terms k , $\mathbf{s}'$, $\mathbf{s}$ and w change and the other matrix terms remain constant. The consequences over the boundary condition matrix concern elements that depend on the source, and are: τ_{ss} , $\boldsymbol{\tau}_{sd}$, $\boldsymbol{\rho}_{sd}$, ρ_{so} and τ_{so} . Thus, to allow their estimation, an explicit dependency of the boundary terms on the scattering ones has to be accomplished:

$$\{\tau_{ss} \Rightarrow \tau_{ss}(k), \boldsymbol{\tau}_{sd} \Rightarrow \boldsymbol{\tau}_{sd}(k, \mathbf{s}', \mathbf{s}), \boldsymbol{\rho}_{sd} \Rightarrow \boldsymbol{\rho}_{sd}(k, \mathbf{s}', \mathbf{s}), \rho_{so} \Rightarrow \rho_{so}(k, \mathbf{s}', \mathbf{s}, w), \tau_{so} \Rightarrow \tau_{so}(k, \mathbf{s}', \mathbf{s}, w')\}. \quad (88)$$

Moreover, in the discrete leaf case, the hot spot effect is taken into account in the computation of ρ_{so} , in this case it will be noted as ρ_{so}^{HS} (Verhoef, 1998).

To distinguish SAIL++ boundary matrix terms from our model terms, ++ will be added to SAIL++ terms as superscript.

A.3 SAIL++ equation reformulation

In our study, we need to separate the upward diffuse fluxes created by the first collision with leaves of direct flux from the upward fluxes created by multiple collisions, the corresponding radiances are called L_+^1 and L_+^∞ , respectively. Indeed, a specific processing for L_+^1 is proposed in this paper in order to take into account the hot spot effect as well as to conserve energy. As defined, L_+^1 depends on E_s and can be extended when traveling under the vegetation. Compared to L_+ [cf. Eq. (74)], L_+^1 does not increase by L_- and L_+^1 itself scattering. Thus its variation is governed by [cf. Eq. (80)]

$$\frac{dL_+^1(z, \Omega_+)}{dz} = [\mathfrak{s} \circ E_s(z, \Omega_s)](\Omega_+) - [\mathfrak{k} \circ L_+^1(z)](\Omega_+). \quad (89)$$

Now, concerning L_+^∞ , it does not depend any more on E_s . However it increases by L_+^1 , L_- and L_+^∞ itself scattering and decreases, as usual, by extinction. It is given by

$$\frac{dL_+^\infty(z, \Omega_+)}{dz} = [\mathfrak{B}' \circ L_+^1(z)](\Omega_+) + [\mathfrak{B} \circ L_-(z)](\Omega_+) - [\mathfrak{A} \circ L_+^\infty(z)](\Omega_+), \quad (90)$$

According to this decomposition, the reformulation of SAIL++ equation set is as follows. Eq. (74) has to be replaced by Eqs. (89) and (90). In Eqs (73), (75) and (76), L^+ has to be replaced by $L_+^1 + L_+^\infty$. One obtains

$$\frac{dL_-(z, \Omega_-)}{dz} = -[\mathfrak{s}' \circ E_s(z, \Omega_s)](\Omega_-) + [\mathfrak{A} \circ L_-(z)](\Omega_-) - [\mathfrak{B} \circ L_+^1(z)](\Omega_-) - [\mathfrak{B} \circ L_+^\infty(z)](\Omega_-), \quad (91)$$

$$\frac{dE_o^+(z, \Omega_o)}{dz} = wE_s(z, \Omega_s) + [\mathfrak{v} \circ L_-(z)] + [\mathfrak{v}' \circ L_+^1(z)] + [\mathfrak{v}' \circ L_+^\infty(z)] - KE_o^+(z, \Omega_o), \quad (92)$$

$$\frac{dE_o^-(z, \Omega_o)}{dz} = -w'E_s(z, \Omega_s) - [\mathfrak{v}' \circ L_-(z)] - [\mathfrak{v} \circ L_+^1(z)] - [\mathfrak{v} \circ L_+^\infty(z)] + KE_o^-(z, \Omega_o). \quad (93)$$

The reformulated SAIL++ equation set is composed by Eqs. (72), (91), (89), (90) (92) and (93).

B. Vegetation local density

To define a realization of a vegetation distribution within the canopy in the discrete leaf case, Knyazikhin et al. (1998) propose the definition of an indicator function:

$$\chi(\vec{r}) = \begin{cases} 1, & \text{if } \vec{r} \in \text{vegetation,} \\ 0, & \text{otherwise,} \end{cases} \quad (94)$$

where $\vec{r} = (x, y, z)$ is a point within the canopy. Then, they define a fine spatial mesh by dividing the layer into non-overlapping fine cells ($e(\vec{r})$) with volume $V[e(\vec{r})]$. Thus, the foliage area volume density (FAVD) could be defined as follows:

$$u_L(\vec{r}) = \frac{1}{V[e(\vec{r})]} \int_{\vec{t} \in e(\vec{r})} \chi(\vec{t}) d\vec{t}. \quad (95)$$

By defining the average density of leaf area per unit volume, called d_L (depends only on leaf shape and orientation distribution), u_L is written simply as follows

$$u_L(\vec{r}) = d_L \chi(\vec{r}). \quad (96)$$

In a 1-D RT model, we always need an averaged value of u_L , called $\bar{u}_L$, rather than a unique realization. Assuming that we have a number, N_c , of canopy realizations, then

$$\bar{u}_L(\vec{r}) \approx \frac{\sum_{n=1}^{N_c} u_L^{(n)}(\vec{r})}{N_c}, \quad (97)$$

with $u_L^{(n)}$ the value of FAVD for the realization number n . Similarly, we can define the probability of finding foliage in $e(\vec{r})$ called P_χ as follows

$$P_\chi(\vec{r}) = \frac{\sum_{n=1}^{N_c} \chi^{(n)}(\vec{r})}{N_c}, \quad (98)$$

with $\chi^{(n)}$ the indicator function for the realization n . Finally, we obtain

$$\bar{u}_L(\vec{r}) = d_L P_\chi(\vec{r}). \quad (99)$$

C. Virtual flux decomposition validation

In this appendix, we will answer the following questions: why $\forall n \in \mathbb{N}$, L_1^n [cf. Eq. (17)] can be considered a radiance distribution and why the expression of $P_{\chi,n}$ [cf. Eq. (21)] is valid. The validity can be proved if we can show that the derived radiance hemispherical distributions L_- and L_+^∞ , and radiances in observation direction E_o^+ and E_o^- , are correct. Since the proofs are similar, we will show only the validity of E_o^+ expression. As validation reference, we will adopt the AddingSD approach.

Recall that the upward elementary diffuse flux, $d^3 E_+^1$, in an elementary solid angle $d\Omega$, created by the first collision with the vegetation in an elementary volume at point N with thickness dt is given by [cf. Figure 1 and Eq. (14)]

$$\begin{aligned} d^3 E_+^1(N \rightarrow M, \Omega) &= dL_+^1(N \rightarrow M, \Omega) \cos(\theta) d\Omega, \\ &= E_s(0) \exp[(k+K)(t-z)] \exp\left[\frac{\sqrt{kK}}{b}(1 - \exp[-b(z-t)])\right] \\ &\quad \times \exp(kz) \pi^{-1} w(N, \Omega_s \rightarrow \Omega) dt \cos(\theta) d\Omega. \end{aligned} \quad (100)$$

As defined in Section 2.1.3, the a posteriori extinction, K_{HS} , of a flux present on M collided only one time at N and initially coming from a source solid angle Ω_s is (cf. Figure 1)

$$\begin{aligned} K_{HS}(\Omega|\Omega_s, 0, t-z) &= K + \lim_{u \rightarrow z} \frac{1}{b} \frac{\sqrt{kK}(\exp[b(t-u)] - \exp[b(t-z)])}{u-z}, \\ &= K - \sqrt{kK} \exp[-b(z-t)]. \end{aligned} \quad (101)$$

This decrease of extinction value means a decrease in the collision probability locally around M . Thus, in turn, means a decrease in the probability of finding foliage at M , P_χ (cf. Appendix B). Now, according to Eq. (99)

$$\left. \begin{aligned} K &= d_L P_\chi K_0 \\ K_{HS} &= d_L P_{\chi,HS}(\Omega|\Omega_s, 0, t-z) K_0 \end{aligned} \right\} \Rightarrow P_{\chi,HS}(\Omega|\Omega_s, 0, t-z) = \frac{K_{HS}}{K} P_\chi, \quad (102)$$

were K_0 is the normalized extinction parameter corresponding to K [cf. Eq. (77)], $P_{\chi,HS}(\Omega|\Omega_s,0,t-z)$ is the 'a posteriori' probability of finding vegetation at M . To be simpler, it will be noted $P_{\chi,HS}(\Omega|\Omega_s,t-z)$.

The angular differentiation of E_o^+ ($d^3E_o^+(z,\Omega \rightarrow \Omega_o)$) that depends only on $d^3E_+^1$ is

$$\begin{aligned} \frac{d[d^3E_o^+(t \rightarrow z,\Omega \rightarrow \Omega_o)]}{dz} &= w'_{HS}(t \rightarrow z,\Omega \rightarrow \Omega_o)d^3E_+^1(N \rightarrow M,\Omega), \\ &= w'_{HS}(\Omega|\Omega_s,t-z)L_+^1(t \rightarrow z,\Omega)dt \cos(\theta)d\Omega, \end{aligned} \quad (103)$$

where

$$w'_{HS}(\Omega|\Omega_s,t-z) = d_L P_{\chi,HS}(\Omega|\Omega_s,t-z)w'_0(\Omega \rightarrow \Omega_o). \quad (104)$$

Now,

$$\begin{aligned} L_+^1(z,\Omega) &= E_s(0) \exp(kz) \pi^{-1} w(\Omega_s \rightarrow \Omega) \\ &\times \int_{-H}^z \exp[(k+K)(t-z)] \exp\left[\frac{\sqrt{kK}}{b}(1 - \exp[-b(z-t)])\right] dt. \end{aligned} \quad (105)$$

Therefore,

$$\begin{aligned} \frac{d[d^2E_o^+(z,\Omega \rightarrow \Omega_o)]}{dz} &= E_s(0) \exp(kz) \pi^{-1} w(\Omega_s \rightarrow \Omega) \cos(\theta) d\Omega d_L w'_0(\Omega \rightarrow \Omega_o) \\ &\times \int_{-H}^z P_{\chi,HS}(\Omega|\Omega_s,t-z) \exp[(k+K)(t-z)] \\ &\times \exp\left[\frac{\sqrt{kK}}{b}(1 - \exp[-b(z-t)])\right] dt. \end{aligned} \quad (106)$$

Now, it is straightforward to show that

$$\begin{aligned} &P_{\chi,HS}(\Omega|\Omega_s,t-z) \exp[(k+K)(t-z)] \exp\left[\frac{\sqrt{kK}}{b}(1 - \exp[-b(z-t)])\right] \\ &= \sum_{n=0}^{+\infty} P_{\chi,n} A_n (-1)^n \exp[(k+K+nb)(t-z)]. \end{aligned} \quad (107)$$

Then, Eq. (106) becomes

$$\begin{aligned} \frac{d[d^2E_o^+(z,\Omega \rightarrow \Omega_o)]}{dz} &= E_s(0) \exp(kz) \pi^{-1} w(\Omega_s \rightarrow \Omega) \cos(\theta) d\Omega d_L w'_0(\Omega \rightarrow \Omega_o) \\ &\times \int_{-H}^z \sum_{n=0}^{+\infty} P_{\chi,n} A_n (-1)^n \exp[(k+K+nb)(t-z)] dt, \\ &= \sum_{n=0}^{+\infty} A_n (-1)^n E_s(0) \exp(kz) \pi^{-1} w(\Omega_s \rightarrow \Omega) \cos(\theta) d\Omega \\ &\times \int_{-H}^z w'_n(\Omega \rightarrow \Omega_o) \exp[(k+K+nb)(t-z)] dt, \\ &= \sum_{n=0}^{+\infty} A_n (-1)^n w'_n(\Omega \rightarrow \Omega_o) L_+^{1,n}(z,\Omega) \cos(\theta) d\Omega. \end{aligned} \quad (108)$$

Equations (30) and (108) are the same which implies the validity of our approach.

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Our planet is nowadays continuously monitored by powerful remote sensors operating in wide portions of the electromagnetic spectrum. Our capability of acquiring detailed information on the environment has been revolutionized by revealing its inner structure, morphology and dynamical changes. The way we now observe and study the evolution of the Earth's status has even radically influenced our perception and conception of the world we live in. The aim of this book is to bring together contributions from experts to present new research results and prospects of the future developments in the area of geosciences and remote sensing; emerging research directions are discussed. The volume consists of twenty-six chapters, encompassing both theoretical aspects and application-oriented studies. An unfolding perspective on various current trends in this extremely rich area is offered. The book chapters can be categorized along different perspectives, among others, use of active or passive sensors, employed technologies and configurations, considered scenario on the Earth, scientific research area involved in the studies.

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