We are IntechOpen, the world's leading publisher of Open Access books Built by scientists, for scientists

6,300

Open access books available

172,000

International authors and editors

190M

Downloads

Our authors are among the

154

Countries delivered to

TOP 1%

most cited scientists

12.2%

Contributors from top 500 universities

WEB OF SCIENCE™

Selection of our books indexed in the Book Citation Index
in Web of Science™ Core Collection (BKCI)

Interested in publishing with us?
Contact book.department@intechopen.com

Numbers displayed above are based on latest data collected.
For more information visit www.intechopen.com

TOLERANCE ANALYSIS USING JACOBIAN-TORSOR MODEL: STATISTICAL AND DETERMINISTIC APPLICATIONS

Walid Ghie

Université du Québec en Abitibi-Témiscamingue
445, Boul. de l'Université, Rouyn-Noranda, Qc

Canada, J9X 5E4

Walid.Ghie@uqat.ca

ABSTRACT

In industry, the current practice concerning geometrical specifications for mechanical parts is to include both *dimensions* and *tolerances*. The objective of these specifications is to describe a class of functionally acceptable mechanical parts that are geometrically similar. To ensure that their functionality is respected during assembly, designers have to apply tolerance analysis. A model based on either worst-case or statistical type analysis may be used.

This paper explains both types using the Jacobian-Torsor unified model. For statistical tolerance analysis we consider Monte Carlo simulation and for the worst case type we consider arithmetic intervals. Although the numerical example presented is for a three-part assembly, the method used is capable of handling three-dimensional geometry.

KEY WORDS

Tolerance, Jacobian, Torsor, Analysis, Statistical.

1. Introduction

Tolerancing decisions can profoundly impact the quality and cost of products. Thus to ensure that functional requirements for an assembly are respected, the designer determines the dimension chains and definitions for each part using the specifications described for tolerancing. The tolerance values can then be calculated using the worst-case or statistical approach. In our work we analyze how both methods can influence the results obtained, using our Jacobian-torsor model. We also describe the example we used to study these methods.

2. Related Works

Tolerance model variables are derived from a variety of tolerance models, represented either in conventional plus-minus or geometric tolerance formats. The assembly response system noted by functional requirement (FR) may also be represented in two models: closed mathematical and relative positioning. In the closed mathematical model, mathematical equations are formulated and the design function variations calculated through applying the equations directly. In the relative positioning model, an optimization model is used instead of the closed mathematical equations. Using the closed mathematical model, in previous papers a tool for deterministic tolerance analysis was presented [1], [2] which used an interval arithmetic formulation:

$$\begin{bmatrix} [\underline{u}, \bar{u}] \\ [\underline{v}, \bar{v}] \\ [\underline{w}, \bar{w}] \\ [\underline{\alpha}, \bar{\alpha}] \\ [\underline{\beta}, \bar{\beta}] \\ [\underline{\delta}, \bar{\delta}] \end{bmatrix}_{FR} = \left[\begin{bmatrix} J_1 & J_2 & J_3 & J_4 & J_5 & J_6 \end{bmatrix}_{FE1} \cdots \begin{bmatrix} J_1 & J_2 & J_3 & J_4 & J_5 & J_6 \end{bmatrix}_{FEN} \right] \cdot \begin{bmatrix} \begin{bmatrix} [\underline{u}, \bar{u}] \\ [\underline{v}, \bar{v}] \\ [\underline{w}, \bar{w}] \\ [\underline{\alpha}, \bar{\alpha}] \\ [\underline{\beta}, \bar{\beta}] \\ [\underline{\delta}, \bar{\delta}] \end{bmatrix}_{FE1} \\ \cdots \cdots \cdots \\ \begin{bmatrix} [\underline{u}, \bar{u}] \\ [\underline{v}, \bar{v}] \\ [\underline{w}, \bar{w}] \\ [\underline{\alpha}, \bar{\alpha}] \\ [\underline{\beta}, \bar{\beta}] \\ [\underline{\delta}, \bar{\delta}] \end{bmatrix}_{FEN} \end{bmatrix} \tag{1}$$

Where:

$\begin{bmatrix} [\underline{u}, \bar{u}] \\ \vdots \\ [\underline{\delta}, \bar{\delta}] \end{bmatrix}_{FR}$: Small displacement torsors were associated with some functional requirements (play, gap, clearance) represented as a [FR] vector or some Functional Element uncertainties (tolerance, kinematic link, etc.) were also represented as [FE] vectors; where N represents the number of torsors in a kinematic chain;

and $\begin{bmatrix} [\underline{u}, \bar{u}] \\ \vdots \\ [\underline{\delta}, \bar{\delta}] \end{bmatrix}_{FEi}$

$\begin{bmatrix} J_1 \cdots J_6 \end{bmatrix}_{FEi}$: A Jacobian matrix expresses a geometrical relation between a [FR] vector and some corresponding [FE] vector;

$\underline{u}, \underline{v}, \underline{w}, \underline{\alpha}, \underline{\beta}, \underline{\delta}$

:

Lower limits of $u, v, w, \alpha, \beta, \delta$;

$\bar{u}, \bar{v}, \bar{w}, \bar{\alpha}, \bar{\beta}, \bar{\delta}$

:

Upper limits of $u, v, w, \alpha, \beta, \delta$.

In this work, the SDT or *Small displacement torsor with interval* scheme was adopted to represent future deviation and the *Jacobian matrix* has suggested for mapping all SDT in a dimensional chain. The following section describes these elements.

Small Displacement torsor with interval: The concept of the small displacement torsor (SDT) was developed in the seventies by P. Bourdet and A. Clément, in order to solve the general problem of fitting a geometrical surface model to a set of points. In its first form this concept was largely used in the field of metrology. In tolerancing we are more interested in surface or feature (axis, center, plane) variations relative to the nominal position. The components of these variations can be represented by a screw parameter, and this screw parameter is then called a small displacement screw. It may be used directly in its generic form to represent potential variations along and about all three Cartesian axes [3].

Described in [4] is an inventory of all standard tolerance zones, along with their corresponding torsor representations and geometrical constraints. For a given functional element, a torsor represents its various possible dispersions in translation (u, v, w) and in rotation (α, β, δ) as opposed to its remaining degrees of freedom (represented here by zeros). The following table shows the various classes of tolerance zones, their corresponding torsors and their constraints, as suggested by Desrochers and adapted, with minor changes, from [1,2].

ZONE PLANAR

Constraints	Torsor

<div><div>Positional constraints</div><div>$-\frac{t}{2} \leq w \leq +\frac{t}{2}$</div><div>Angular constraints</div><div>$-\frac{t}{L_1} \leq \alpha \leq +\frac{t}{L_1}$</div><div>$-\frac{t}{L_2} \leq \beta \leq +\frac{t}{L_2}$</div></div>	<div><div><div><div>$\begin{bmatrix} \underline{u}, \bar{u} \\ \underline{v}, \bar{v} \\ \underline{w}, \bar{w} \\ \underline{\alpha}, \bar{\alpha} \\ \underline{\beta}, \bar{\beta} \\ \underline{\delta}, \bar{\delta} \end{bmatrix}_{FEi}$</div><div>$= \begin{bmatrix} \begin{bmatrix} 0,0 \\ 0,0 \end{bmatrix} \\ \begin{bmatrix} -\frac{t}{2}, +\frac{t}{2} \end{bmatrix} \\ \begin{bmatrix} -\frac{t}{L_1}, +\frac{t}{L_1} \end{bmatrix} \\ \begin{bmatrix} -\frac{t}{L_2}, +\frac{t}{L_2} \end{bmatrix} \\ \begin{bmatrix} 0,0 \end{bmatrix} \end{bmatrix}_{FEi}$</div></div></div></div>
--	--

Table 1. Small Displacement torsor with interval

Jacobian matrix: The purpose of the Jacobian matrix is to express the relation between the small displacement of all functional elements (FE) and the functional requirement (FR) sought. In this matrix therefore the columns are extracted from the various homogeneous transform matrices T_0^i relating the functional element (FE) reference frames to that of the functional requirement (FR).

$$T_0^i = \begin{bmatrix} [R_{3 \times 3}]_0^i & \vdots & [P_{3 \times 1}]_0^i \\ \dots & \vdots & \dots \\ 0 & \vdots & 1 \end{bmatrix} = \begin{bmatrix} \vec{C}_{1i} & \vec{C}_{2i} & \vec{C}_{3i} & \vec{d}_i \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4 \times 4} \tag{2}$$

Where

- $R_0^i = \begin{bmatrix} \vec{C}_{1i} & \vec{C}_{2i} & \vec{C}_{3i} \end{bmatrix}$

: These vectors represent the orientation of reference frame i with respect to 0, where the columns $\vec{C}_{1i}$, $\vec{C}_{2i}$ and $\vec{C}_{3i}$ respectively indicate the unit vectors along the axes X_i , Y_i and Z_i of reference mark i in reference mark 0.
- $\vec{d}_i = \begin{bmatrix} dx_i & dy_i & dz_i \end{bmatrix}^T$

: Position vector defining the origin for the reference frame i in 0.

The Jacobian matrix is formulated by:

$$\begin{bmatrix} J_1 & \dots & J_6 \end{bmatrix}_{FEi} = \begin{bmatrix} \begin{bmatrix} R_0^i \end{bmatrix}_{3 \times 3} & \vdots & \begin{bmatrix} W_i^n \end{bmatrix}_{3 \times 3} \bullet \begin{bmatrix} R_0^i \end{bmatrix}_{3 \times 3} \\ \dots\dots\dots & \vdots & \dots\dots\dots\dots\dots\dots \\ \begin{bmatrix} 0 \end{bmatrix}_{3 \times 3} & \vdots & \begin{bmatrix} R_0^i \end{bmatrix}_{3 \times 3} \end{bmatrix}_{6 \times 6} \tag{3}$$

Where $\begin{bmatrix} W_i^n \end{bmatrix}_{3 \times 3}$ is a *Skew-symmetric* matrix [5] allowing the representation of the vector $\begin{bmatrix} \vec{d}_n - \vec{d}_i \end{bmatrix}$ with $dx_i^n = dx_n - dx_i$, $dy_i^n = dy_n - dy_i$ and $dz_i^n = dz_n - dz_i$ knowing that $\vec{d}_n$ and $\vec{d}_i$ can be obtained from the transformation matrix in equation (2). $\begin{bmatrix} W_i^n \end{bmatrix}_{3 \times 3} \bullet \begin{bmatrix} R_0^i \end{bmatrix}_{3 \times 3}$ should be used to directly obtain the first three elements in the fourth, fifth and sixth columns of the Jacobian matrix. $\begin{bmatrix} R_0^i \end{bmatrix}_{3 \times 3}$ represent the orientation matrix of reference frame i relative to 0 (from equation (2)).

The Jacobian-torsor model can be expressed as follows:

$$\begin{bmatrix} FR \end{bmatrix} = \begin{bmatrix} J \end{bmatrix} \begin{bmatrix} FE_i \end{bmatrix} \quad (4)$$

Where

- $\begin{bmatrix} FR \end{bmatrix}$: 6×1 small displacements torsor of the functional requirement; 6×1 small torsor displacements for the functional requirement;
- $\begin{bmatrix} J \end{bmatrix}$: $6 \times n$ Jacobian matrix;
- $\begin{bmatrix} FE_i \end{bmatrix}$: 6×1 individual small displacements torsors of each part in the chain, $i = 1$ to n ; and n represent is the total number of functional elements in the chain;

As shown, column matrix $\begin{bmatrix} FR \end{bmatrix}$ represents the dispersions around a given functional condition where the six small displacements are bounded by interval values. Similarly, the corresponding column matrix $\begin{bmatrix} FE_i \end{bmatrix}$ represents the various functional elements encountered in the tolerance chain where intervals are again used to represent variations on each element. Naturally, the terms in this expression remain the same as those used in “conventional” Jacobian modeling [15, 16].

Thereafter we build on this model through applying the statistical analyses (Monte Carlo) and deterministic analysis (Worst-Case).

3. Analysis tolerances

In this method we apply deterministic and statistical tolerance analysis in order to compare the results. In our research, tolerance analysis consists of an assembly simulation with manufactured parts, *i.e.* parts with geometric variations. Deterministic and statistical tolerance analyses make use of a relationship expressed by Equation (5) where Y is the assembly response (gap or functional characteristics) and $X = \{x_1, x_2, \dots, x_N\}$ the values of certain characteristics (such as situation deviations and/or intrinsic deviations) of the individual parts or subassemblies making up the assembly. Function f is the assembly response function. This relationship can be expressed in any form, in which a value for Y given values of X may be computed. An explicit analytic expression or an implicit analytic

expression could be used, or the process could involve complex engineering calculations, conducting experiments or running simulations.

$$Y = f\left(x_1, x_2, \dots, x_N\right) \tag{5}$$

Where :

- $x_1, x_2, \dots, x_N$: Parameters include dimensional chain tolerances
- Y : Represent the functional requirement [FR];
- f : A geometric expression relates the nominal dimension to the assembly's functionality.

3.1 Tolerances analysis deterministic

Generally in tolerance the application of functional analysis involves the entering of tolerances for all parts $x_1, x_2, \dots, x_N$ involved in the dimensional chain, for those ratings related to the functional requirements. As output, we obtain the arithmetic value of functional requirement Y . Thus, when we want to apply an analysis it is assumed we will have a predetermined set of dimensional tolerances. Moreover, this analysis considers the worst possible combinations of individual tolerances and then examines the functional characteristics. In this approach the arithmetic intervals used have the highest possible maximum values [1, 2, 6].

In the deterministic analysis, we will apply the Jacobian-Torsor model (Equation (5) based on equation (1)). The application of this model is based on the arithmetic interval [1, 2, 6, 7].

3.2 Statistical analysis tolerances

Same of WC, statistical tolerance analysis uses a relationship of the form (Equation (5) based on equation (1)). In this case, the input variables $X = \{x_1, x_2, \dots, x_N\}$ are continuous random variables.

A variety of methods and techniques allow to estimate the probability distribution of Y and the probability of the respect of the geometrical requirement. Essentially, the methods can be categorized into four classes according to the different type of function f [8-10]: Linear propagation (Root sum of squares), Non-linear propagation (Extended Taylor series), Numerical integration (Quadrature technique). Monte Carlo simulation.

So, in the case of the statistical tolerance analysis, the function Y is not available in analytic form, the determination of the value of Y involve the running simulation. Therefore, we use a Monte Carlo simulation. Indeed, Monte Carlo technique is easily the most popular tool used in tolerancing problems [9-10]. *Monte Carlo simulation* is a method for *iteratively* evaluating a deterministic model using sets of random numbers as inputs. This method is often used when the model is complex, nonlinear, or involves more than just a couple uncertain parameters.

Statistical tolerance analysis, is based on a very simple thinking: Random number generators are used to generate a sample of numbers $x_1, x_2, \dots, x_N$, belonging to the random variables $X_1, X_2, \dots, X_N$, respectively. The value of $Y, y_1 = f(x_1, x_2, \dots, x_N)$, corresponding to this sample is computed. This procedure is replicated a large number of samples. This would yield a random sample $\{x_1, x_2, \dots, x_N\}$ for Y .

Fig. 1. Monte Carlo simulation

Clearly, if statistical tolerances are specified for the inputs (set of $(x_1, x_2, \dots, x_N)$), a statistical tolerance can be calculated for the output (set of $(y_1, y_2, \dots, y_N)$). This amounts to determining the average and standard deviation of the output. Simulations can always be used to predict these two values. However, there also exist a variety of methods for deriving approximate and sometimes exact equations for the average and standard deviation. Details can be found in Taylor [11, 12] in these approaches have a variety of names including statistical tolerance analysis, propagation of errors and variation transmission analysis.

Returning to the case where $Y = f(x_1, \dots, x_n)$, equations can be derived for the average and standard deviation of Y in terms of the average and standard deviation of the x_i 's.

Based on uncertain propagation theory [11, 12], the statistical tolerance for Y (*in our context Y is FR_D*) can then be calculated as follows:

$$FR_D = FR_{Dm} \pm \delta FR_D$$

(6)

Where:

- FR_D
- :
- Represents the functional requirement in direction D , with D is x, y or z. In this paper, we centred only on the direction in translation, and did not evaluate the rotational direction.
- FR_{Dm}
- :
- Best value of FR or average of FR.
- δFR_D
- :
- Deviation of functional requirement, statistical is represented by standard deviation squared.

Next section shows by an example these methods.

4. Numerical example

The *centering pin mechanism* in Figures 2 to 5 was used to demonstrate the use of this tool. In these figures, we labelled some key tolerances from t_a to t_e . The mechanism featured three parts, with two functional conditions as shown in Figures 2, 3 and 4. The purpose of this example is to explore the functional condition FR1 (Figure 5).

Fig. 2. Pin for centering pin mechanism

Fig. 3. Base for centering pin mechanism

Fig. 4. Block for centering pin mechanism

The figure below shows the assembly drawing.

Fig. 5. Detail assembly for centering pin mechanism with two FRs

In this example, the designer specified an FR1 of $\pm 0.5\text{mm}$. (For more information, see the analysis for this example described in [14, 15]). To meet this objective the designer proposed a list of tolerances from ta to tg , as described in the table below.

For this simple example, the relevant parameters are as follows:

Tolerances values proposed							
ta	0.2	tc	0.2	te	$H11 : 0.00/0.13$	tg	0.1
tb	0.1	td	0.1	tf	$h8 : - 0.033/0.000$		

Table 2. Assigned tolerances values

As shown in Figure 6, the contact surfaces between the parts are first identified. A connection graph is then constructed for this mechanism in order to establish the dimensional chain around the functional condition FR1.

The resulting kinematic chain contains three internal pairs (FE0, FE1), (FE2, FE3), (FE4, FE5) as well as one kinematic pair (FE1, FE2). Note that there are two FRs: FR1 applies between (FE0, FE5) and FR2 between (FE3, FE4) and that defined for functional fit 20 is H11/h8.

In this example, we assumed that the reference frames are in the middle of the tolerance or contact uncertainty zone, and are associated with the second element in the pairs defined above. The kinematic torsor for (FE1, FE2) is considered null because the contact between the two planes is assumed perfect, and form tolerances are not being considered here.

Fig. 6. Kinematic chain identification

The torsor calculation method accounts for the effect that might result when a tolerance dimension or position is imposed simultaneously on the same surface. The tables 3 and 4 list the constraints of torsors and their details that will be included in the Jacobian-Torsor model. From this, the final expression for the Jacobian-Torsor model and its intervals becomes:

$$\begin{bmatrix} [u,\bar{u}] \\ [v,\bar{v}] \\ [w,\bar{w}] \\ [\alpha,\bar{\alpha}] \\ [\beta,\bar{\beta}] \\ [\delta,\bar{\delta}] \end{bmatrix}_{FRr} = \begin{bmatrix} 1 & 0 & 0 & 0 & -55 & 100 \\ 0 & 1 & 0 & 55 & 0 & 0 \\ 0 & 0 & 1 & -100 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}_{FE1} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 75 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -75 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}_{FE2} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 75 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -75 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}_{FE3} \begin{bmatrix} 1 & 0 & 0 & 0 & 45 & 0 \\ 0 & 1 & 0 & -45 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}_{FE4} \begin{bmatrix} [0,0] \\ [0,0] \\ [\pm 0.05] \\ [\pm 0.002] \\ [\pm 0.0025] \\ [0,0] \end{bmatrix}_{FE1} \begin{bmatrix} [\pm 0.05] \\ [0,0] \\ [\pm 0.05] \\ [\pm 0.004] \\ [0,0] \\ [\pm 0.004] \end{bmatrix}_{FE2} \begin{bmatrix} [\pm 0.0815] \\ [0,0] \\ [\pm 0.0815] \\ [\pm 0.0033] \\ [0,0] \\ [\pm 0.0033] \end{bmatrix}_{FE3} \begin{bmatrix} [\pm 0.05] \\ [0,0] \\ [\pm 0.05] \\ [\pm 0.0033] \\ [0,0] \\ [\pm 0.0033] \end{bmatrix}_{FE4} \quad (7)$$

The torsor components corresponding to undetermined elements in the kinematic pair can then be replaced by null elements, because they do not affect small displacements along the analysis direction (*w* in this case, on axis *Z*₀). Thus, the calculated functional condition becomes. By using the approach described in [1, 2], we obtain the following deterministic method: Along the *Z* direction, **FR_Z = ±0.976 mm**.

FE#	Constraints
FE1	$w = \pm (ta - tb) / 2$ $\alpha = \pm ta / 100; \beta = \pm ta / 80$
FE2	$u = \pm (tc - td) / 2; w = \pm (tc - td) / 2$ $\alpha = \pm tc / 50; \delta = \pm tc / 50$
FE3 ¹	$t = ES - ei \begin{cases} u = \pm t / 2; w = \pm t / 2 \\ \alpha = \pm t / 50; \delta = \pm t / 50 \end{cases}$
FE4	$u = \pm tg / 2; w = \pm tg / 2$ $\alpha = \pm tg / 30; \delta = \pm tg / 30$

Table 3. Constraints details

¹ FE3 : contact element between elements 3 and 4

FE#	FE1	FE2	FE3 ²	FE4
Torsors form	$\begin{bmatrix} [0,0] \\ [0,0] \\ [\underline{w},\overline{w}] \\ [\underline{\alpha},\overline{\alpha}] \\ [\underline{\beta},\overline{\beta}] \\ [0,0] \end{bmatrix}_{FE1}$	$\begin{bmatrix} [\underline{u},\overline{u}] \\ [0,0] \\ [\underline{w},\overline{w}] \\ [\underline{\alpha},\overline{\alpha}] \\ [0,0] \\ [\underline{\delta},\overline{\delta}] \end{bmatrix}_{FE2}$	$\begin{bmatrix} [\underline{u},\overline{u}] \\ [0,0] \\ [\underline{w},\overline{w}] \\ [\underline{\alpha},\overline{\alpha}] \\ [0,0] \\ [\underline{\delta},\overline{\delta}] \end{bmatrix}_{FE3}$	$\begin{bmatrix} [\underline{u},\overline{u}] \\ [0,0] \\ [\underline{w},\overline{w}] \\ [\underline{\alpha},\overline{\alpha}] \\ [0,0] \\ [\underline{\delta},\overline{\delta}] \end{bmatrix}_{FE4}$
Torsors values	$\begin{bmatrix} [0,0] \\ [0,0] \\ [\pm 0.1] \\ [\pm 0.003] \\ [\pm 0.0025] \\ [0,0] \end{bmatrix}_{FE1}$	$\begin{bmatrix} [\pm 0.05] \\ [0,0] \\ [\pm 0.05] \\ [\pm 0.004] \\ [0,0] \\ [\pm 0.004] \end{bmatrix}_{FE2}$	$\begin{bmatrix} [\pm 0.065] \\ [0,0] \\ [\pm 0.065] \\ [\pm 0.0026] \\ [0,0] \\ [\pm 0.0026] \end{bmatrix}_{FE3}$	$\begin{bmatrix} [\pm 0.05] \\ [0,0] \\ [\pm 0.05] \\ [\pm 0.0033] \\ [0,0] \\ [\pm 0.0033] \end{bmatrix}_{FE4}$

Table 4. Torsor details

In the statistical approach, the random variables generated using last column (Equation (7)). Following this generation and based on the principle described in Section 3, the dispersion based on this condition becomes functional (where D represents the direction along the axe Z):

$$FR_D = FR_{Dm} \pm \delta FR_D = \underline{FR_Z} = 0.00 \pm 0.27 \text{ mm.}$$

Fig. 7. Functional requirement distribution

² FE3 : contact element between elements 3 and 4

We can then see that the statistical dispersion is smaller than that of the deterministic method because:

- The deterministic method assumes that mistakes happen all at once and are most probable. By contrast, the statistical method assumes that the maximum values are generated (distribution is normally low) around the average values (in our case the average is zero);
- The best estimate functional condition is null, due to the fact that the JT model hides ratings in its nominal Jacobian matrix. The results are therefore variations around the nominal. In this way we need to extract the face value of the functional condition, which can be done by using the homogeneous matrix transformation between the first and last repository of the ratings string. This work is the subject of current research.
- We can conclude that the tolerances imposed (Table 2) may be expanded to ensure that manufacturing costs will be cheaper, even when we work with the deterministic method the results obtained for the condition are very close to the designer's limits. The statistical method enables us to enhance tolerance, but it does not provide any indication of the maximum expansion possible.
- From the figure 7, *FR* has a normally distribution: In probability theory, if $C=A+B$, if A and B are independent random variables and identically distributed random variables that are normally distributed, then C is also normally distributed. [16].

5. Conclusion

Tolerance analysis is an important step in design. Proper tolerancing ensures that parts will behave as analyzed for stress and deflection. Worst-case tolerancing tends to overestimate output variations, resulting in extra costs when output variations are overestimated. Statistical tolerancing tends to underestimate the output variations, and quality suffers when output variations are underestimated. By using process tolerancing, output behaviour may be accurately predicted, thus providing the desired quality at a lower cost.

We are currently working on a process that will provide expansion coefficients for a tolerancing method situated between the statistical and deterministic methods. In practice this should enable industry designers and fabrication personnel to handle tolerancing in a more effective manner.

Acknowledgements

The authors would like to acknowledge the Université du Québec en Abitibi-Témiscaming (UQAT) for their financial contribution to this ongoing project.

6. References

- [1] W. Ghie, Modèle unifié Jacobien-Torseur pour le tolérancement assisté par ordinateur, *Ph.D. thesis*; Université de Sherbrooke, Canada, 2004.
- [2] A. Desrochers, W. Ghie, L. Laperriere, Application of a unified Jacobian-Torsor model for tolerance analysis, *Journal of Computing and Information Science in Engineering*, 1(3), 2003, pp. 2-14.
- [3] P. Bourdet, A., A. Clément, Study of optimal-criteria identification based on the small displacement screw model, *Annals of the CIRP*, 1988, vol. 37, pp.503-506.
- [4] A. Desrochers, Modeling three dimensional tolerance zones using screw parameters, *CD-ROM Proceedings of ASME 25th Design Automation Conference*, 1999, Las Vegas.
- [5] L. Tsai « *the mechanics of serial and parallel manipulators* », New York, N.Y. : J. Wiley and Sons, ISBN 0-471-32593-7,1999.
- [6] L. Jaulin, M. Kieffer and O. Didrit « *Applied Interval Analysis With Examples in Parameter and State Estimation, Robust Control and Robotics* », World Scientific Publishing Company Incorporated ,ISBN 1852332190, 2001
- [7] R. Moore « *Methods and Applications of Interval Analysis*», Society for Industrial and Applied Mathematic, ISBN 0898711614, 1979.
- [8] L. Laperrière, W. Ghie and A. Desrochers,« *Statistical and deterministic Tolerance Analysis and synthesis Using a unified jacobian-torsor model*», 52nd General Assembly CIRP, San Sebastian, v. 51/1/2002 pp. 417-420.
- [9] L. Laperrière, T. Kabore, Monte-Carlo simulation of tolerance synthesis equations, *International Journal of Production Research*, 11(39), 2001, pp. 2395-2406.
- [10] Bjørke ø, «Computer-Aided Tolerancing», 2nd edition New York: ASME Press; 1989.
- [11] Taylor J. R., Incertitudes et analyse des erreurs dans les mesures physiques avec exercices corrigés, collection Masson Sciences, Dunod, Paris, 2000.
- [12] Taylor, B., Kuyatt, C., «Guidelines for Evaluating and Expressing the Uncertainty of NIST Measurement Results, NIST Technical Note 1297, Gaithersburg, MD: National Institute of Standards and Technology, September, 1994.
- [13] Zhang C., Luo J. and Wang B. Statistical tolerance synthesis using distribution function zones. *International Journal of Production Research* 1999; 37:3995-4006.
- [14] W. Ghie, , A. Desrochers, L. Laperriere, Assessment of design parameter space using tolerance analysis, *10th International Seminar On Computer Aided Tolerancing*, (CIRP,2007, 21-23 March,, Neuburg, Germany.
- [15] W. Ghie, L. Laperriere, A. Desrochers, Re-design of Mechanical Assemblies using the Jacobian-Torsor model, *Models for Computer Aided Tolerancing in Design and Manufacturing*, 9th CIRP International Seminar on Computer-Aided Tolerancing, held at Arizona State, J. K. Davidson , Springer-Verlag GmbH (Eds), 2006, ISBN: 1-4020-5437-8, PP. 95-104.
- [16] William W. Hines, Douglas C. Montgomery, David M. Goldsman, Connie M. Borror, *Probabilités et statistique pour ingénieurs* (Chenelière éducation, 2005). ISBN 2-7651-0245-7.

Modeling Simulation and Optimization - Tolerance and Optimal Control

Edited by Shkelzen Cakaj

ISBN 978-953-307-056-8

Hard cover, 304 pages

Publisher InTech

Published online 01, April, 2010

Published in print edition April, 2010

Parametric representation of shapes, mechanical components modeling with 3D visualization techniques using object oriented programming, the well known golden ratio application on vertical and horizontal displacement investigations of the ground surface, spatial modeling and simulating of dynamic continuous fluid flow process, simulation model for waste-water treatment, an interaction of tilt and illumination conditions at flight simulation and errors in taxiing performance, plant layout optimal plot plan, atmospheric modeling for weather prediction, a stochastic search method that explores the solutions for hill climbing process, cellular automata simulations, thyristor switching characteristics simulation, and simulation framework toward bandwidth quantization and measurement, are all topics with appropriate results from different research backgrounds focused on tolerance analysis and optimal control provided in this book.

How to reference

In order to correctly reference this scholarly work, feel free to copy and paste the following:

Walid Ghie (2010). Tolerance Analysis Using Jacobian-Torsor Model: Statistical and Deterministic Applications, Modeling Simulation and Optimization - Tolerance and Optimal Control, Shkelzen Cakaj (Ed.), ISBN: 978-953-307-056-8, InTech, Available from: <http://www.intechopen.com/books/modeling-simulation-and-optimization-tolerance-and-optimal-control/tolerance-analysis-using-jacobian-torsor-model-statistical-and-deterministic-applications>

INTECH
open science | open minds

InTech Europe

University Campus STeP Ri
Slavka Krautzeka 83/A
51000 Rijeka, Croatia
Phone: +385 (51) 770 447
Fax: +385 (51) 686 166
www.intechopen.com

InTech China

Unit 405, Office Block, Hotel Equatorial Shanghai
No.65, Yan An Road (West), Shanghai, 200040, China
中国上海市延安西路65号上海国际贵都大饭店办公楼405单元
Phone: +86-21-62489820
Fax: +86-21-62489821

© 2010 The Author(s). Licensee IntechOpen. This chapter is distributed under the terms of the [Creative Commons Attribution-NonCommercial-ShareAlike-3.0 License](https://creativecommons.org/licenses/by-nc-sa/3.0/), which permits use, distribution and reproduction for non-commercial purposes, provided the original is properly cited and derivative works building on this content are distributed under the same license.

IntechOpen

IntechOpen